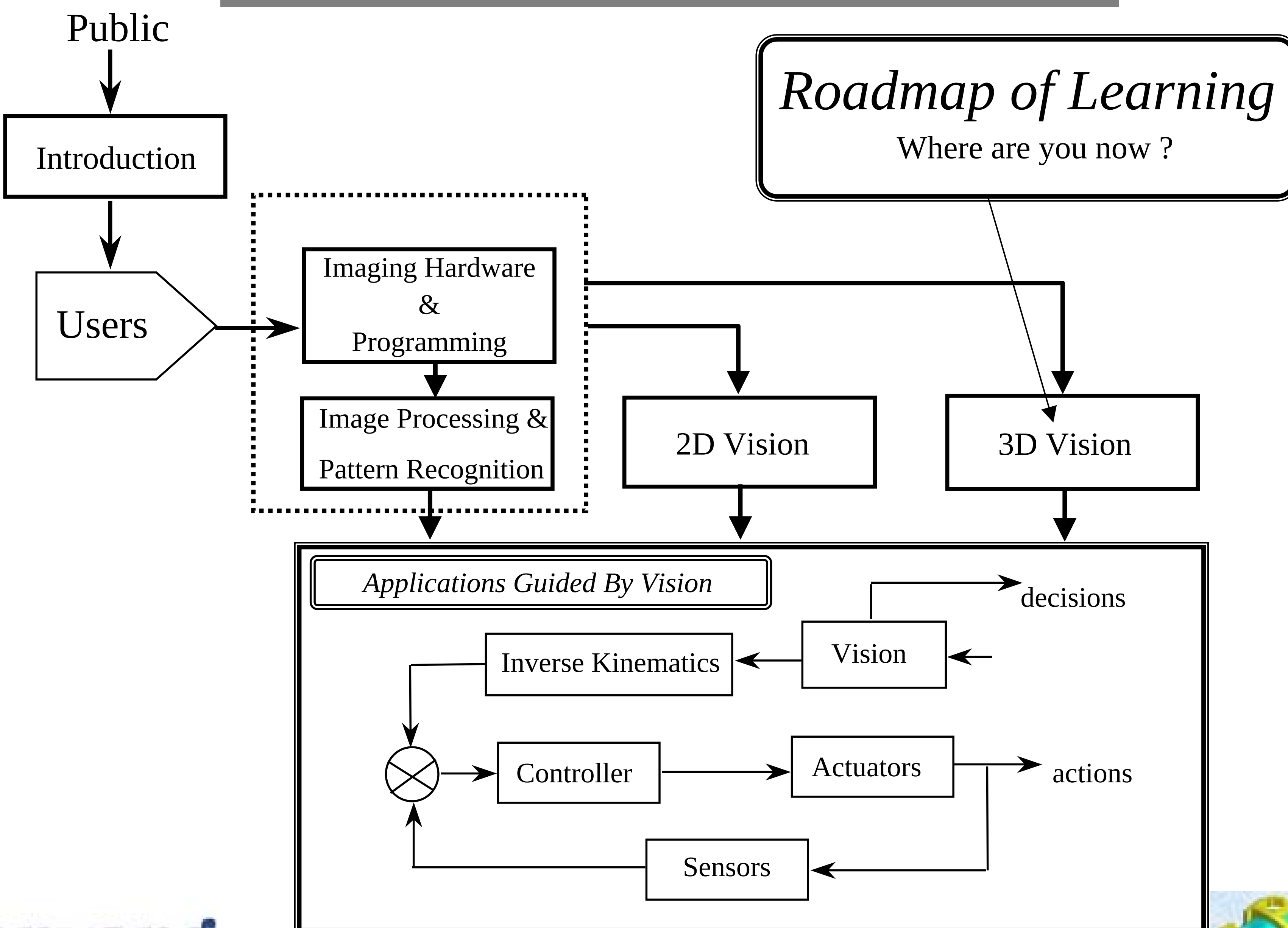

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Imaging Hardware & Programming
Image Processing & Pattern Recognition
2D Vision
3D Vision
Roadmap of Learning
Where are you now ?
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Vision
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Actuators
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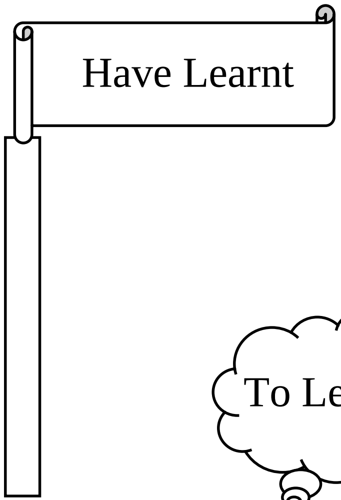
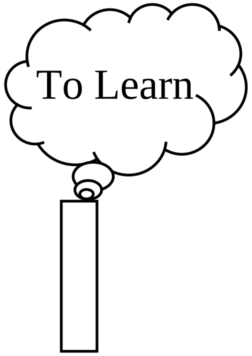
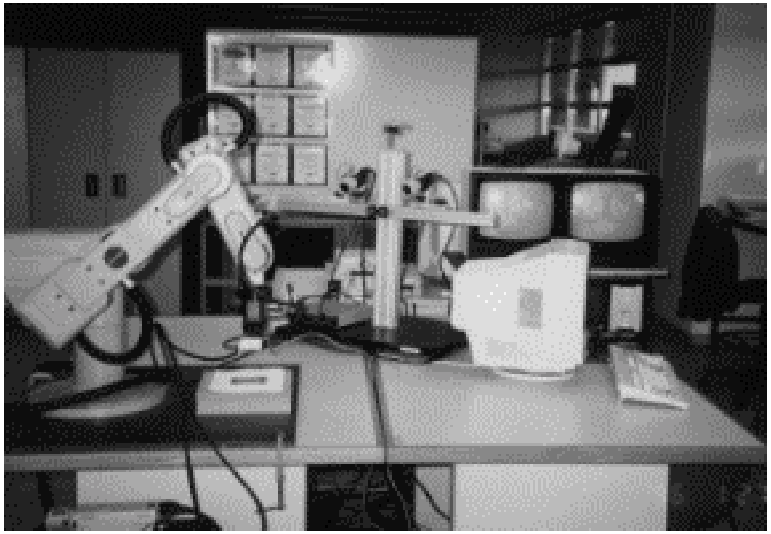
Sensors

CONTENT

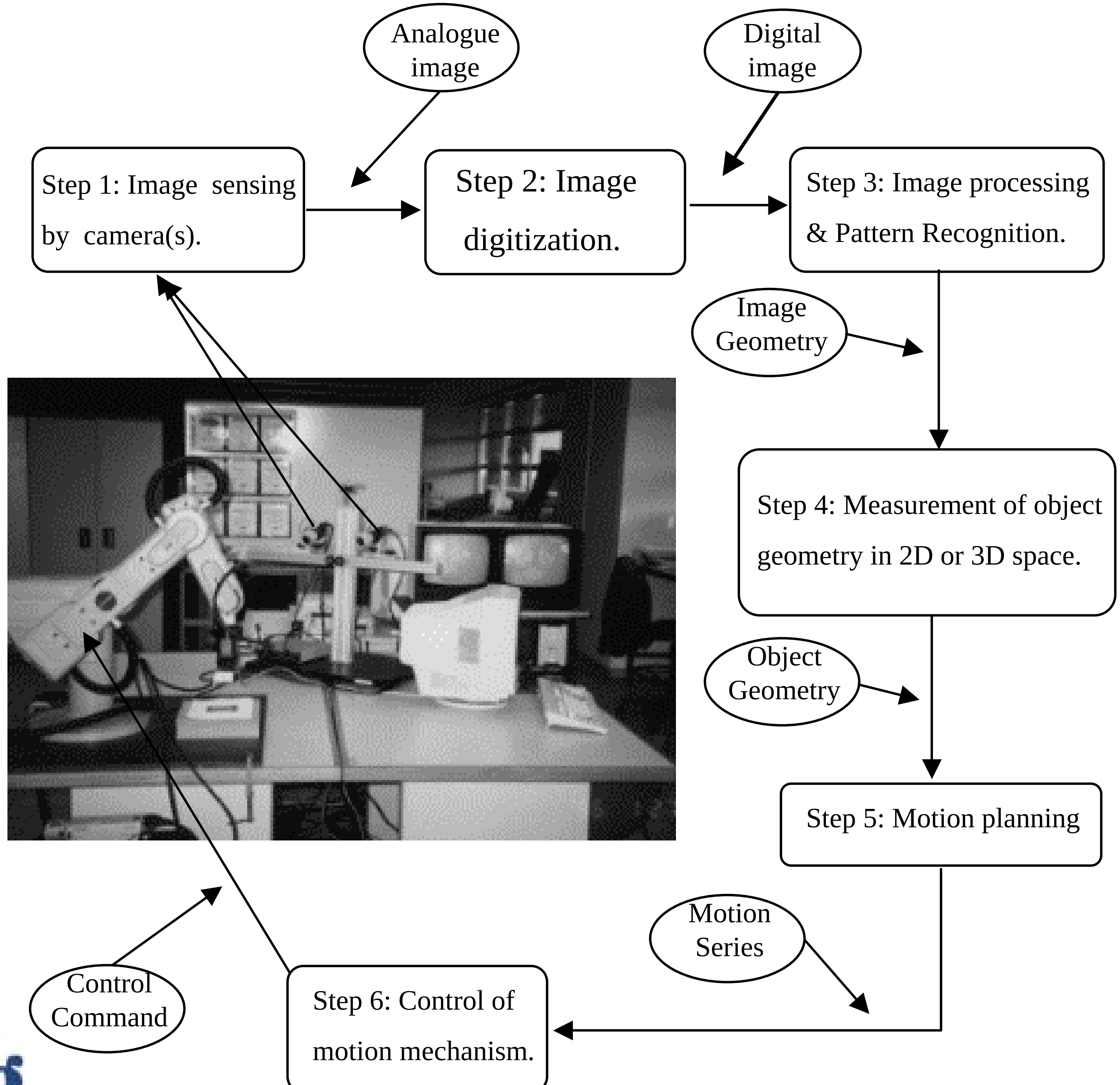




How does a machine vision system work for the task of visual guidance or measurement ?



Answer:


Analogue image
Digital image
Step 1: Image sensing by camera(s).
Step 2: Image digitization.
Step 3: Image processing & Pattern Recognition.
Image Geometry
Step 4: Measurement of object geometry in 2D or 3D space.
Object Geometry
Step 5: Motion planning
Motion Series
Step 6: Control of motion mechanism.
Control Command

How to determine the geometry of an object in a 3D space ? (A Review)

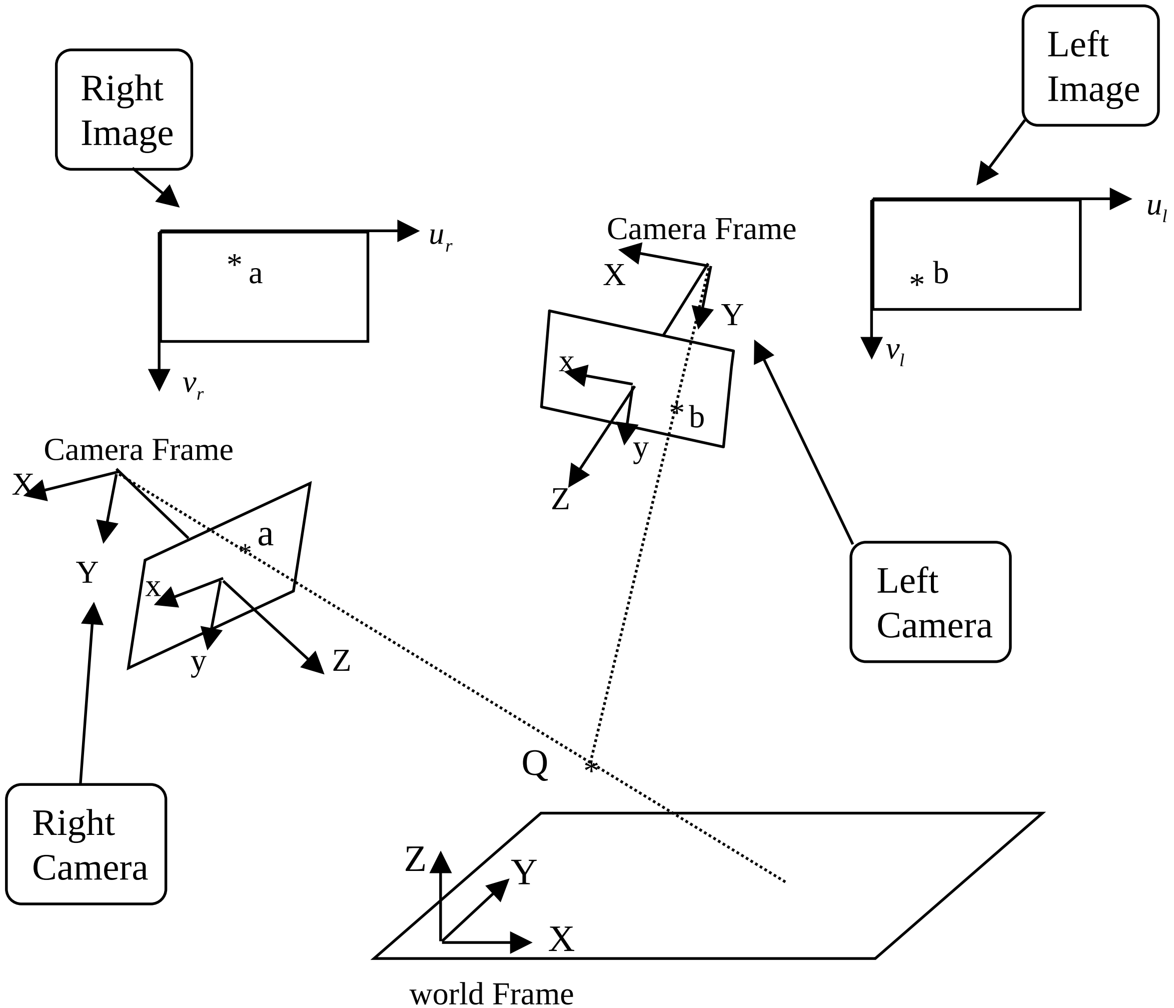
ANSWER:

Possible solutions:

- Asynchronous Methods:
  - Laser Range Finder.
  - 3D Scanner.
  - Motion stereo vision.
- Synchronous Methods:
  - Binocular stereo vision.
  - Structured lighting.

Binocular Stereo Vision

1. System Set-up:



Binocular Stereo Vision

2. Principle:

Step 1: Add in one more camera to form a stereo vision.

Step 2: Use the two cameras to capture a pair of images:
the left image and the right image.

Step 3: The 3D object coordinates in the world frame can be
determined by a simple triangulation.

Binocular Stereo Vision

3. Mathematical Description:

* Input:

Left image coordinates: (u_l, v_l)

Right image coordinates: (u_r, v_r)

3D calibration matrices: $M_l = \{m_{li}, i = 1,2,...,12\}$

$M_r = \{m_{ri}, i = 1,2,...,12\}$

* Output:

3D coordinates: $({}^rX, {}^rY, {}^rZ)$

* Solution:

$$\begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = (A^t \bullet A)^{-1} \bullet (A^t \bullet B).$$

with:

$$A = \begin{pmatrix} (m_{l1} - m_{l9} \bullet u_l) & (m_{l2} - m_{l10} \bullet u_l) & (m_{l3} - m_{l11} \bullet u_l) \\ (m_{l5} - m_{l9} \bullet v_l) & (m_{l6} - m_{l10} \bullet v_l) & (m_{l7} - m_{l11} \bullet v_l) \\ (m_{r1} - m_{r9} \bullet u_r) & (m_{r2} - m_{r10} \bullet u_r) & (m_{r3} - m_{r11} \bullet u_r) \\ (m_{r5} - m_{r9} \bullet v_r) & (m_{r6} - m_{r10} \bullet v_r) & (m_{r7} - m_{r11} \bullet v_r) \end{pmatrix}$$

$$B = \begin{pmatrix} u_l - m_{l4} \\ v_l - m_{l8} \\ u_r - m_{r4} \\ v_r - m_{r8} \end{pmatrix}$$

Proof:

Step 1: The 3D coordinates of an object point and its image coordinates are related to each other by a 3D calibration matrix (see Lecture 16):

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = M_{3\times 4} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix} \quad (1)$$

Step 2: From the left camera, we have:

$$\begin{pmatrix} s \bullet u_l \\ s \bullet v_l \\ s \end{pmatrix} = \begin{bmatrix} m_{l1} & m_{l2} & m_{l3} & m_{l4} \\ m_{l5} & m_{l6} & m_{l7} & m_{l8} \\ m_{l9} & m_{l10} & m_{l11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix} \quad (2)$$

or in algebraic form:

$$\begin{cases} s \bullet u_l = m_{l1} \bullet {}^rX + m_{l2} \bullet {}^rY + m_{l3} \bullet {}^rZ + m_{l4} \\ s \bullet v_l = m_{l5} \bullet {}^rX + m_{l6} \bullet {}^rY + m_{l7} \bullet {}^rZ + m_{l8} \\ s = m_{l9} \bullet {}^rX + m_{l10} \bullet {}^rY + m_{l11} \bullet {}^rZ + 1 \end{cases} \quad (3)$$

The elimination of the scale “s” yields:

$$\begin{cases} (m_{l1} - m_{l9} \bullet u_l) \bullet {}^rX + (m_{l2} - m_{l10} \bullet u_l) \bullet {}^rY + (m_{l3} - m_{l11} \bullet u_l) \bullet {}^rZ = u_l - m_{l4} \\ (m_{l5} - m_{l9} \bullet v_l) \bullet {}^rX + (m_{l6} - m_{l10} \bullet v_l) \bullet {}^rY + (m_{l7} - m_{l11} \bullet v_l) \bullet {}^rZ = v_l - m_{l8} \end{cases} \quad (4)$$

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Step 3: From the right camera, we have:

$$\begin{pmatrix} s \bullet u_r \\ s \bullet v_r \\ s \end{pmatrix} = \begin{bmatrix} m_{r1} & m_{r2} & m_{r3} & m_{r4} \\ m_{r5} & m_{r6} & m_{r7} & m_{r8} \\ m_{r9} & m_{r10} & m_{r11} & 1 \end{bmatrix} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix} \quad (5)$$

or in algebraic form:

$$\begin{cases} s \bullet u_r = m_{r1} \bullet {}^rX + m_{r2} \bullet {}^rY + m_{r3} \bullet {}^rZ + m_{r4} \\ s \bullet v_r = m_{r5} \bullet {}^rX + m_{r6} \bullet {}^rY + m_{r7} \bullet {}^rZ + m_{r8} \\ s = m_{r9} \bullet {}^rX + m_{r10} \bullet {}^rY + m_{r11} \bullet {}^rZ + 1 \end{cases} \quad (6)$$

The elimination of the scale "s" yields:

$$\begin{cases} (m_{r1} - m_{r9} \bullet u_r) \bullet {}^rX + (m_{r2} - m_{r10} \bullet u_r) \bullet {}^rY + (m_{r3} - m_{r11} \bullet u_r) \bullet {}^rZ = u_r - m_{r4} \\ (m_{r5} - m_{r9} \bullet v_r) \bullet {}^rX + (m_{r6} - m_{r10} \bullet v_r) \bullet {}^rY + (m_{r7} - m_{r11} \bullet v_r) \bullet {}^rZ = v_r - m_{r8} \end{cases} \quad (7)$$

Step 4: The combination of Equation (4) and Equation (7) yields:

$$\begin{cases} (m_{l1} - m_{l9} \bullet u_l) \bullet {}^rX + (m_{l2} - m_{l10} \bullet u_l) \bullet {}^rY + (m_{l3} - m_{l11} \bullet u_l) \bullet {}^rZ = u_l - m_{l4} \\ (m_{l5} - m_{l9} \bullet v_l) \bullet {}^rX + (m_{l6} - m_{l10} \bullet v_l) \bullet {}^rY + (m_{l7} - m_{l11} \bullet v_l) \bullet {}^rZ = v_l - m_{l8} \\ (m_{r1} - m_{r9} \bullet u_r) \bullet {}^rX + (m_{r2} - m_{r10} \bullet u_r) \bullet {}^rY + (m_{r3} - m_{r11} \bullet u_r) \bullet {}^rZ = u_r - m_{r4} \\ (m_{r5} - m_{r9} \bullet v_r) \bullet {}^rX + (m_{r6} - m_{r10} \bullet v_r) \bullet {}^rY + (m_{r7} - m_{r11} \bullet v_r) \bullet {}^rZ = v_r - m_{r8} \end{cases} \tag{8}$$

Step 5: Write Equation (8) into a matrix form:

$$A \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \end{pmatrix} = B. \tag{9}$$

with:

$$A = \begin{pmatrix} (m_{l1} - m_{l9} \bullet u_l) & (m_{l2} - m_{l10} \bullet u_l) & (m_{l3} - m_{l11} \bullet u_l) \\ (m_{l5} - m_{l9} \bullet v_l) & (m_{l6} - m_{l10} \bullet v_l) & (m_{l7} - m_{l11} \bullet v_l) \\ (m_{r1} - m_{r9} \bullet u_r) & (m_{r2} - m_{r10} \bullet u_r) & (m_{r3} - m_{r11} \bullet u_r) \\ (m_{r5} - m_{r9} \bullet v_r) & (m_{r6} - m_{r10} \bullet v_r) & (m_{r7} - m_{r11} \bullet v_r) \end{pmatrix}$$

$$B = \begin{pmatrix} u_l - m_{l4} \\ v_l - m_{l8} \\ u_r - m_{r4} \\ v_r - m_{r8} \end{pmatrix}$$

Step 6: Multiply A^t to the both sides of Equation (9):

$$(A^t \bullet A) \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = A^t \bullet B. \qquad (10)$$

Step 7: Multiply $(A^t \bullet A)^{-1}$ to both sides of Equation (10):

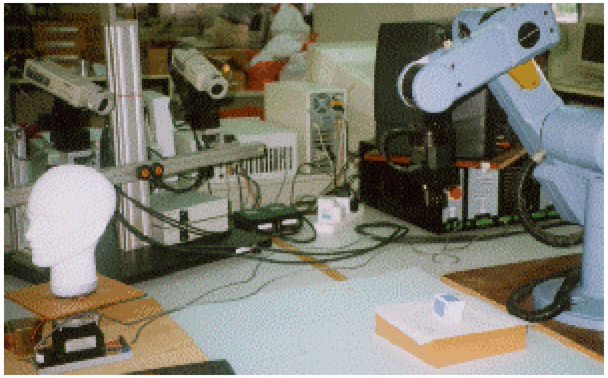
$$\begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \end{pmatrix} = (A^t \bullet A)^{-1} \bullet (A^t \bullet B). \qquad (11)$$

(END OF PROOF)

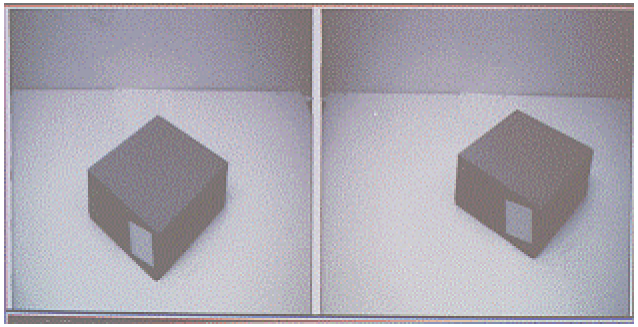
Binocular Stereo Vision

4. Experimental Results:

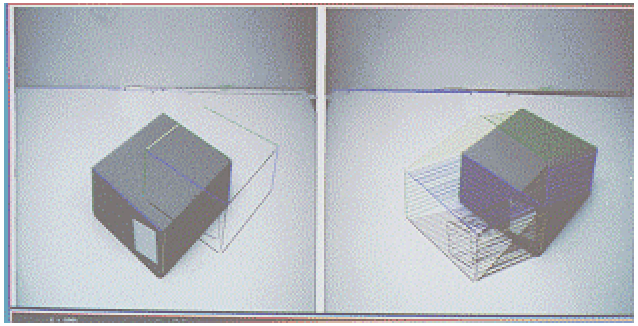
Experimental equipment



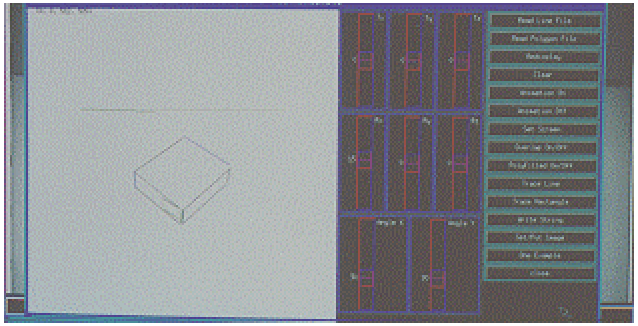
One pair of stereo images



Result of stereo matching

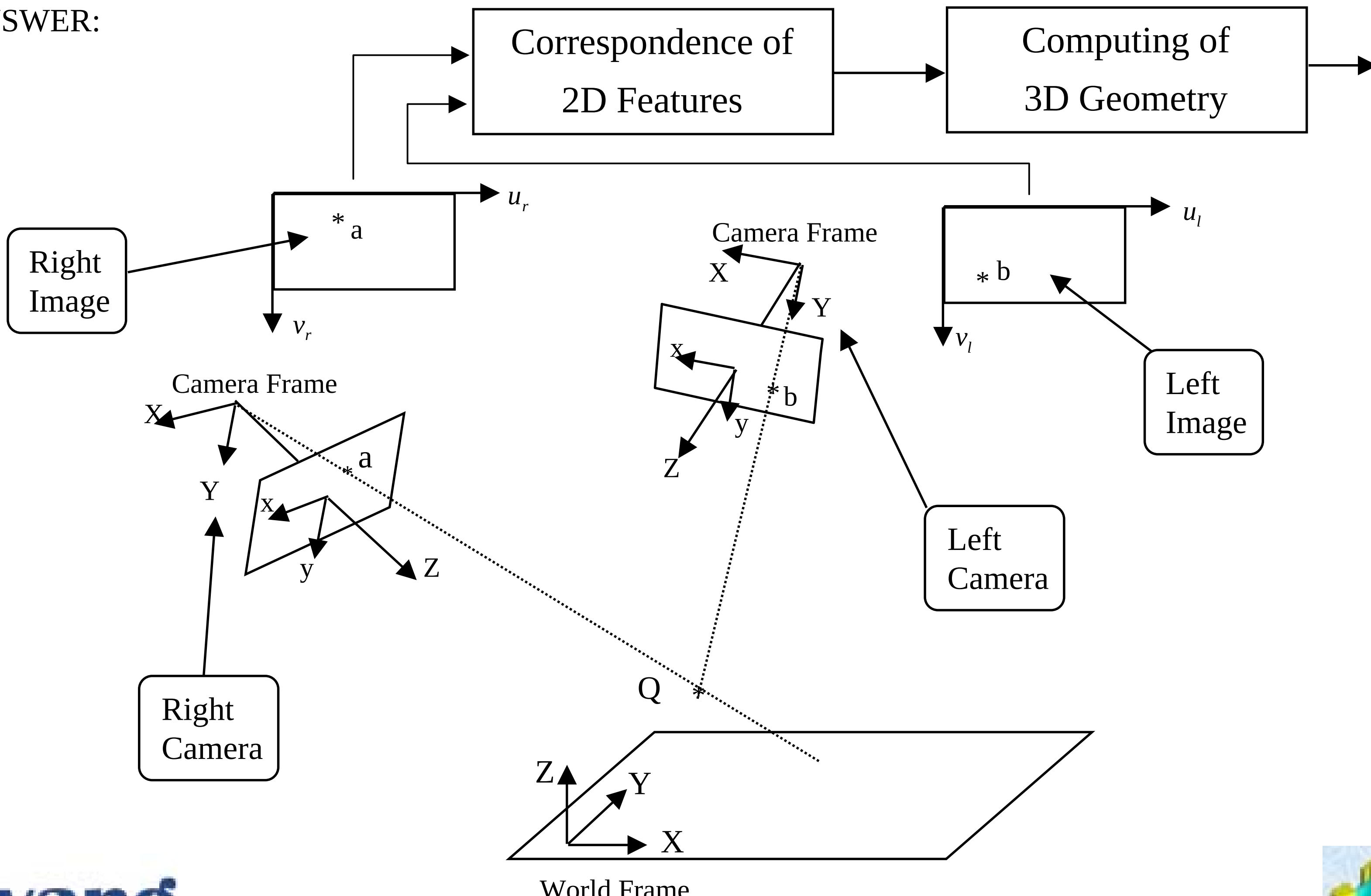


3D view of reconstructed line segments



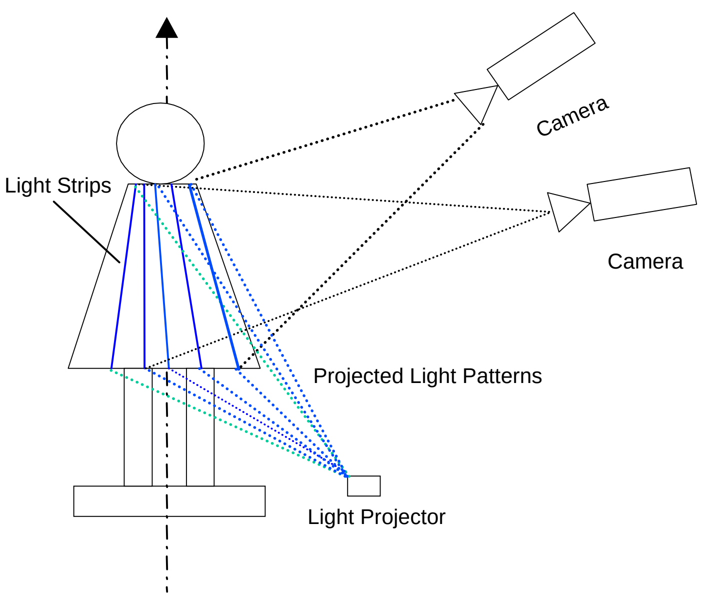
What is the geometric principle of 3D stereo vision ?

ANSWER:



Structured Lighting

1. System Set-up:



Structured Lighting

2. Principle:

Step 1: Project a regular light pattern onto 3D objects.

Step 2: Derive the depth information through:

- the principle of 2D vision if projecting multiple “light” planes.

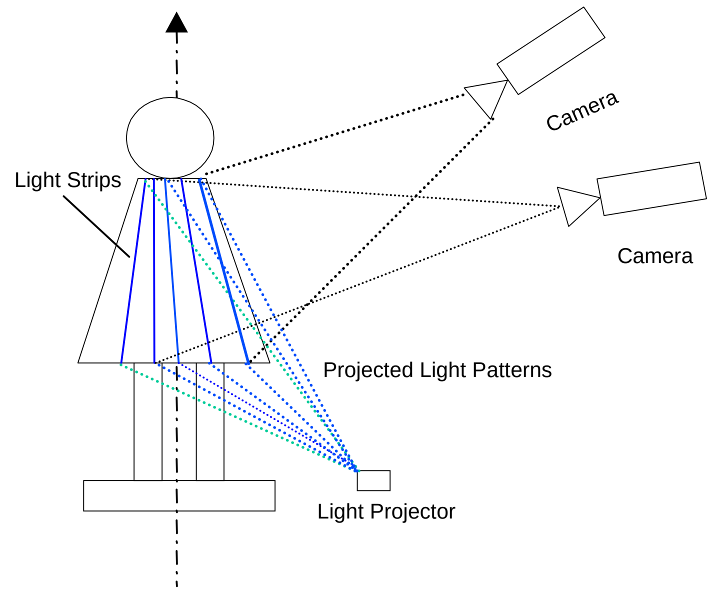
- the deformation of the light patterns caused by the 3D shape of object.

- the principle of binocular stereo vision.

Structured Lighting

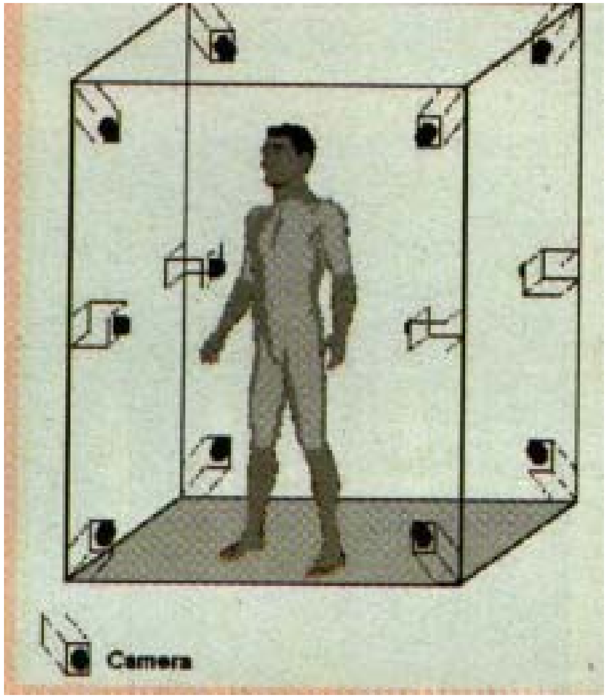
3. Mathematical Description:

Similar to 3D scanner or binocular stereo vision.



Structured Lighting

4. Experimental Results:



SUMMARY

1. In general, it is not possible to compute 3D object coordinates from a single image.

2. By introducing a second camera, a binocular stereo vision is set up.

3. Once we know the image coordinates on both Left and Right images of the binocular stereo vision, the 3D coordinates of an object point can be computed in the following way:

$$\begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \end{pmatrix} = (A^{t} \bullet A)^{-1} \bullet (A^{t} \bullet B).$$

4. If we illuminate the scene by a regular light patterns, we can develop 3D vision system to capture 3D shape of object. This is called “structured lighting” technique. But, the underlying mathematical principle is similar to the one of 3D scanner or binocular stereo vision.

We call the above methods “synchronous methods” because the range or image data acquisition is operated in a parallel way.