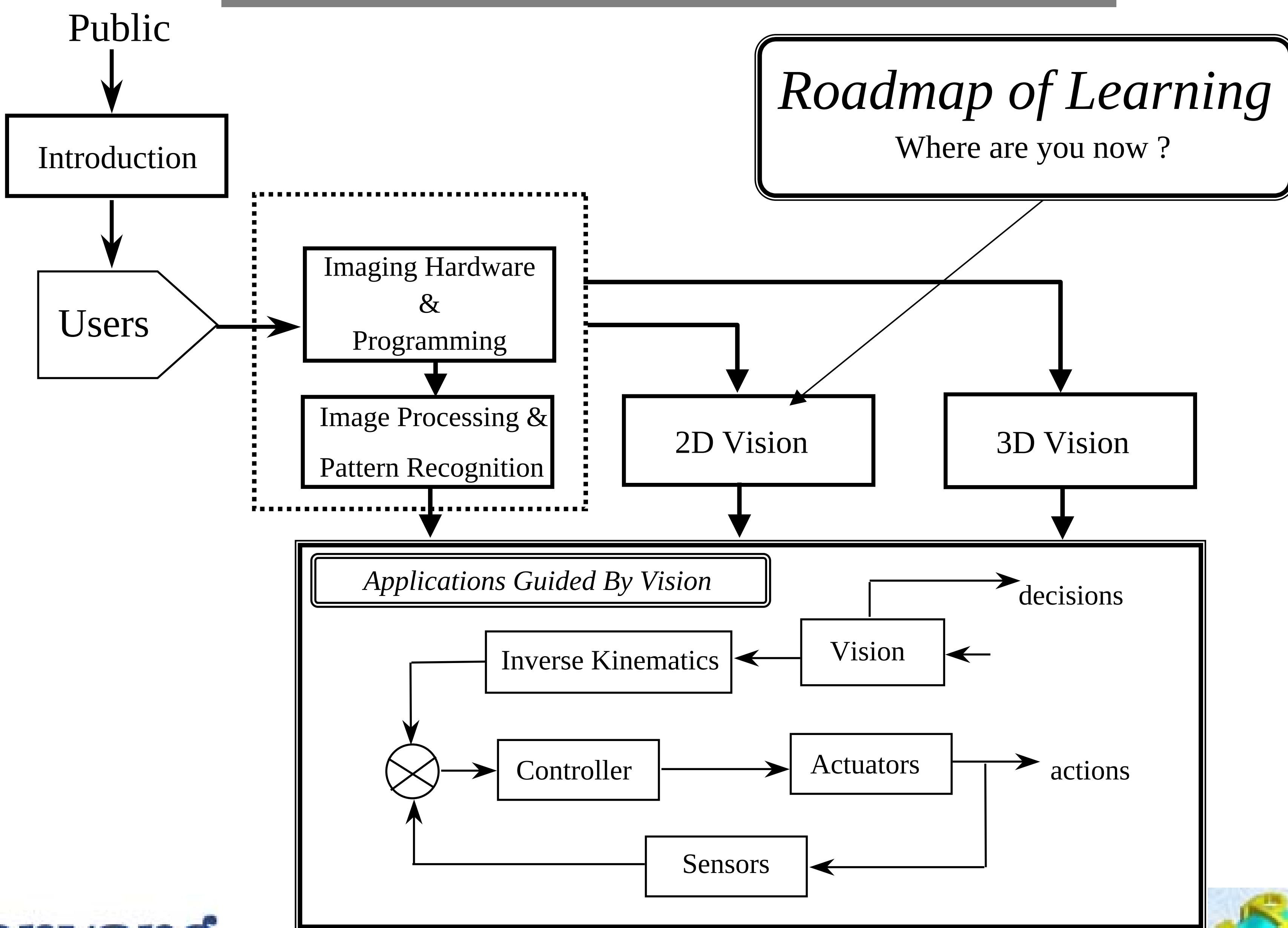

Roadmap of Learning
Where are you now ?
Public
Introduction
Users
Imaging Hardware & Programming
Image Processing & Pattern Recognition
2D Vision
3D Vision
Applications Guided By Vision
decisions
Vision
Inverse Kinematics
Controller
Actuators
actions
Sensors

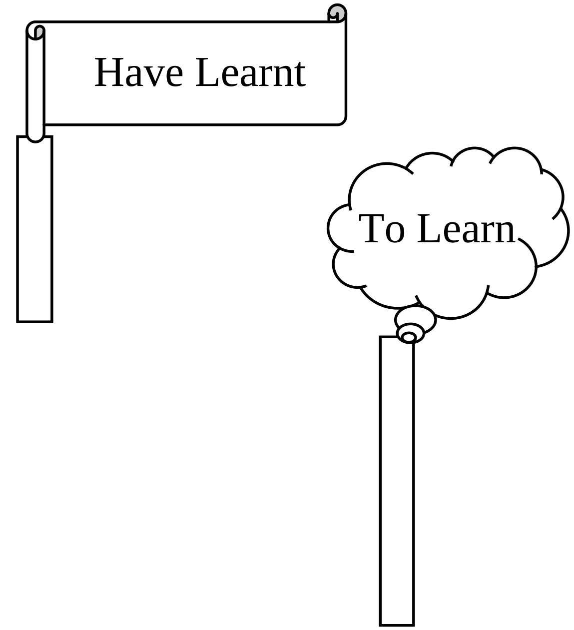
CONTENT

Chapter 7: Fundamentals of 2D Vision

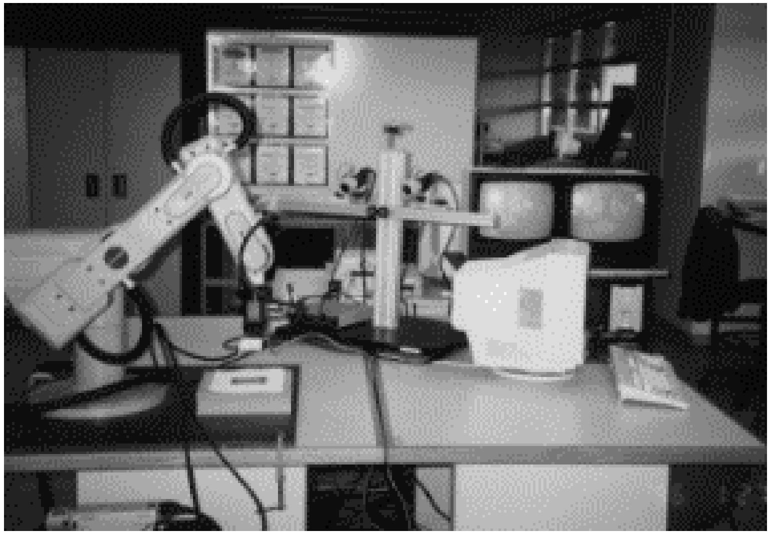
7.1 Geometric Principle of 2D Vision

7.2 Calibration of 2D Vision

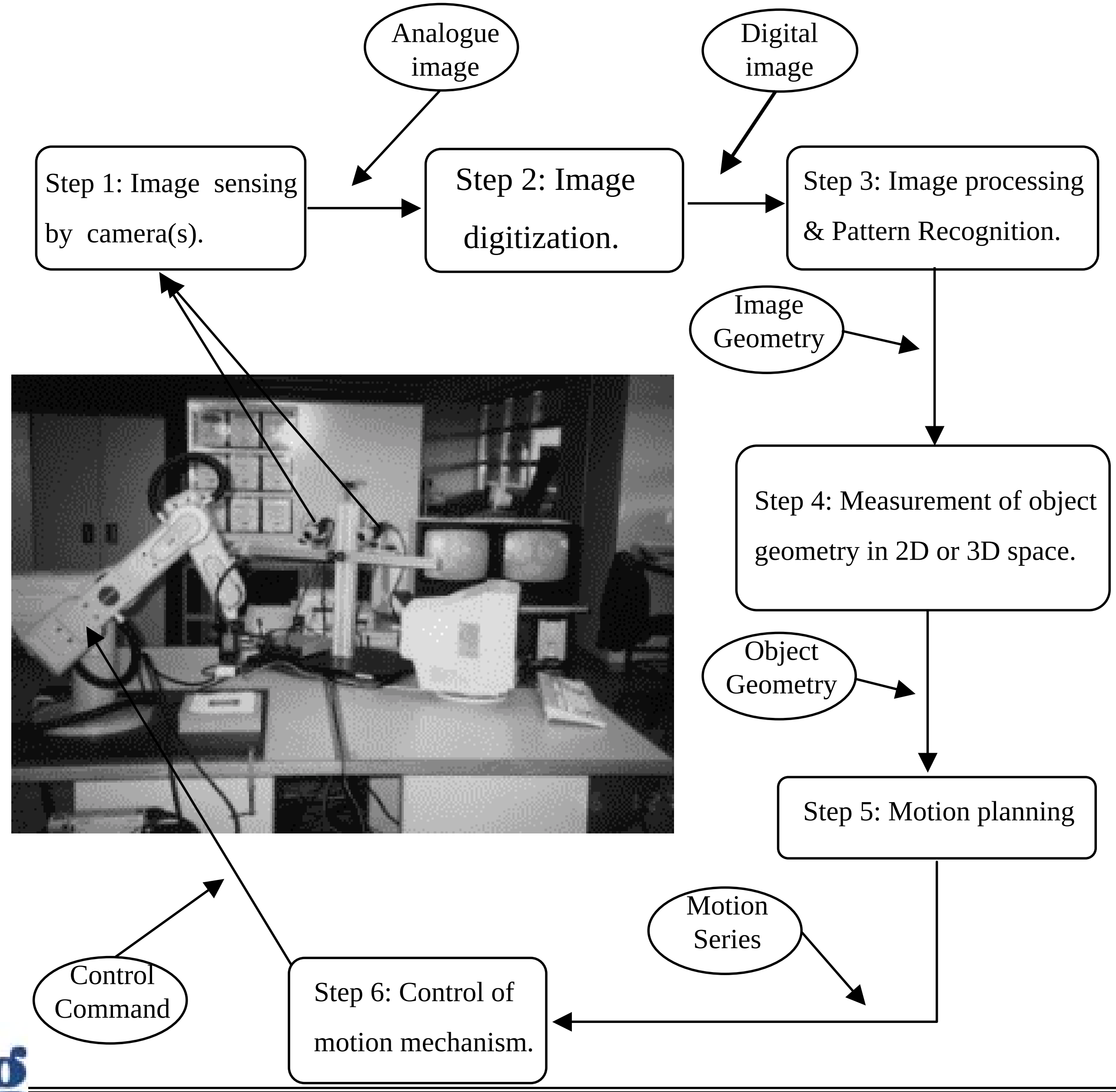
7.3 Measurement of 2D Object



How does a machine vision system work for the task of visual guidance or measurement ?

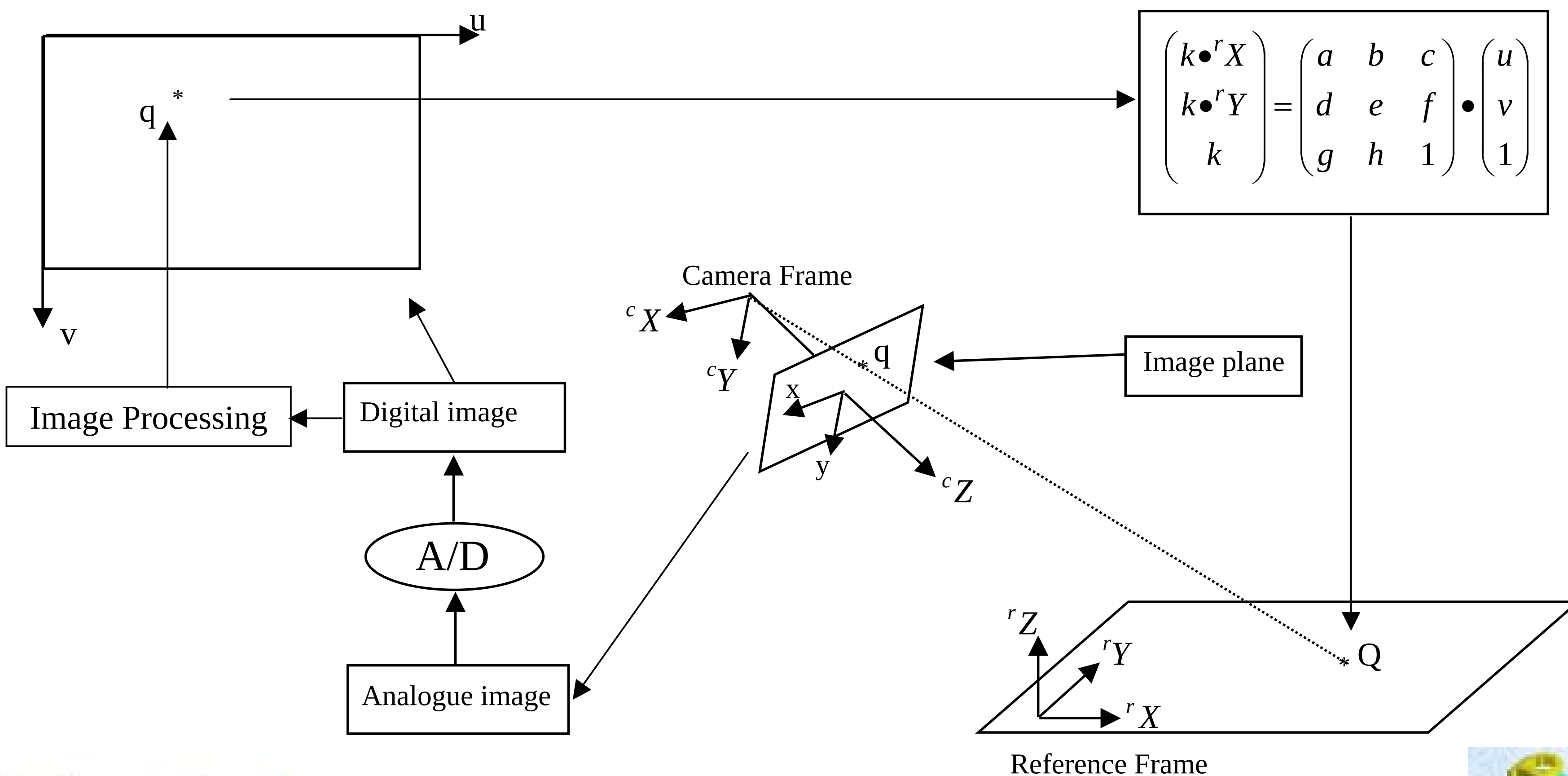


Answer:


Analogue image
Digital image
Step 1: Image sensing by camera(s).
Step 2: Image digitization.
Step 3: Image processing & Pattern Recognition.
Image Geometry
Step 4: Measurement of object geometry in 2D or 3D space.
Object Geometry
Step 5: Motion planning
Motion Series
Step 6: Control of motion mechanism.
Control Command

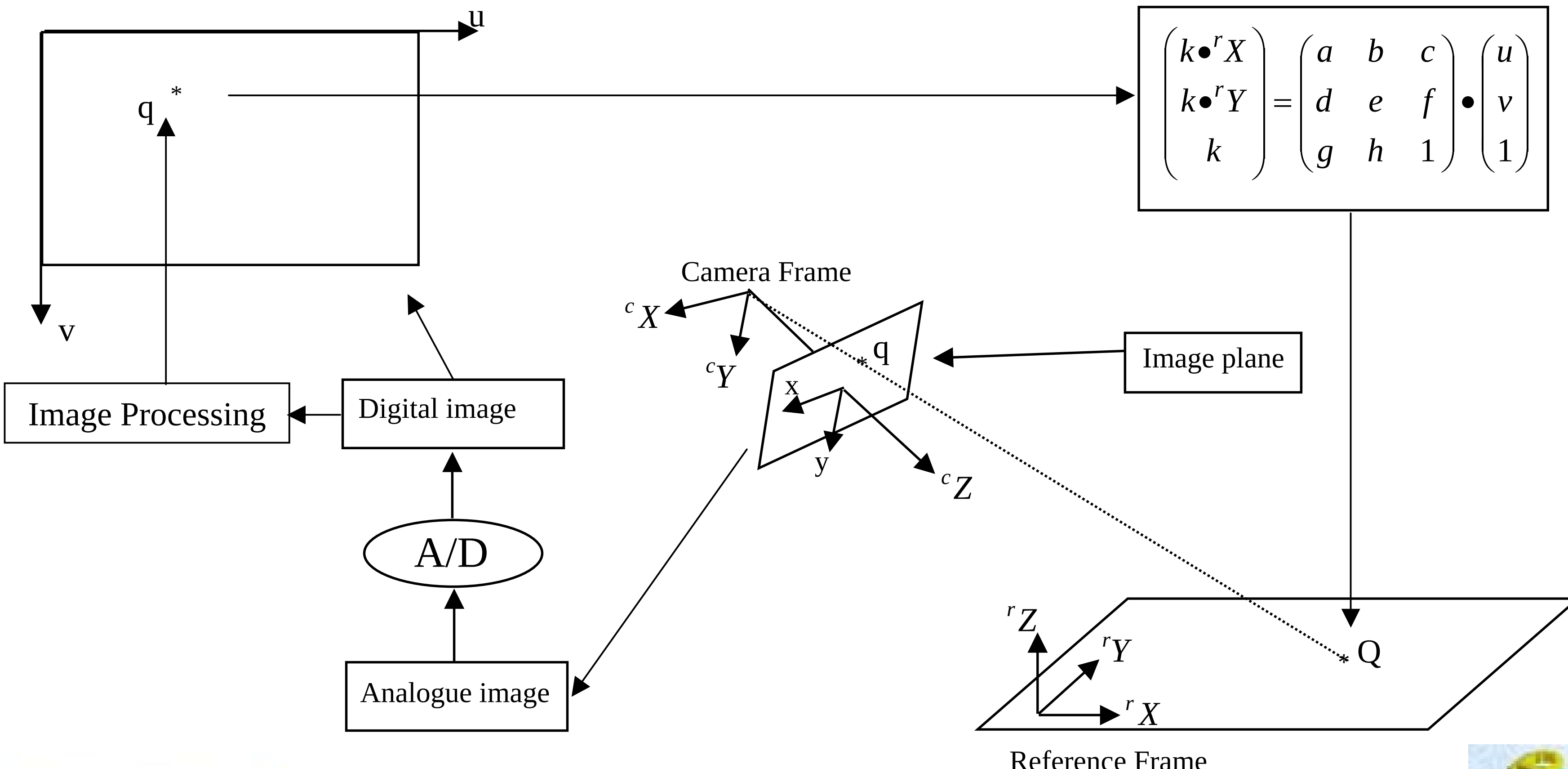
What is the geometric principle of 2D vision ? (A Review)

ANSWER:



Question:

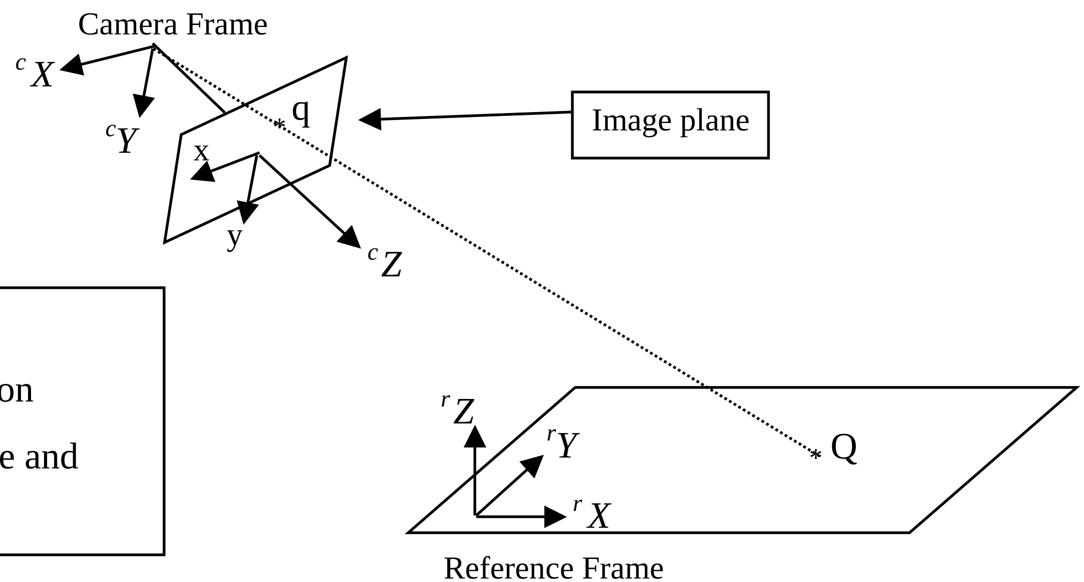
How to determine the coefficients inside the 2D calibration matrix ?



ANSWER:

1. From the proof, it is possible to directly derive the equations of computing the coefficients (a,b,c,d,e,f,g,h).

$$D_{3x3} \propto \{{}^{c}M_{r}, M_{p}, M_{d}\}$$



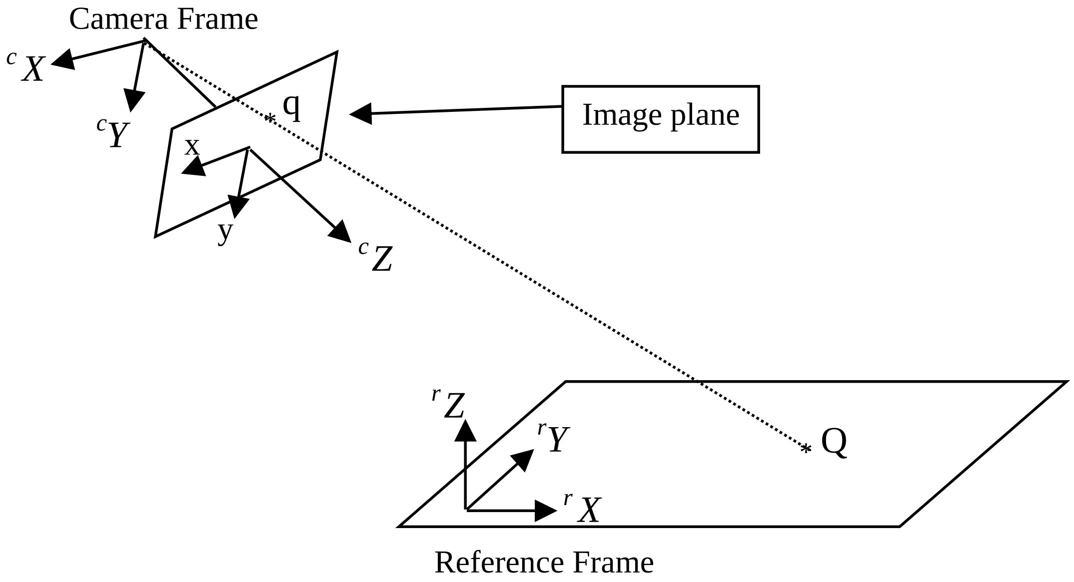
Remark 1:

- The coefficients depend on the motion transformation between object frame and the camera frame.

$$D_{3x3} \propto \{{}^{c}M_r, M_p, M_d\}$$

Remark 2:

- The coefficients also depend on the real dimensions of image pixels.

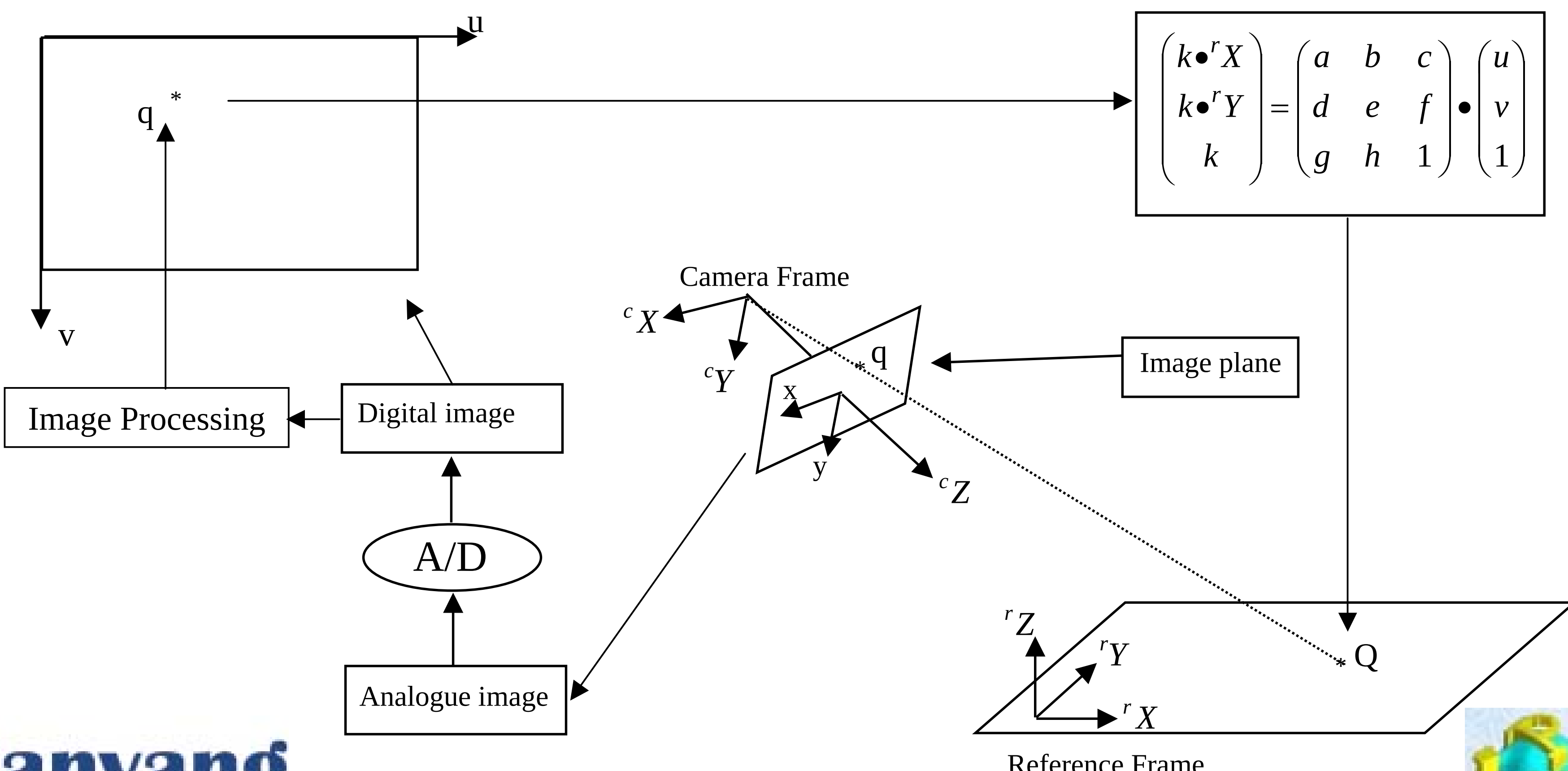


Conclusion:

Direct determination of the coefficients is not practical in real applications !

2. To do Calibration

- Advantages:
  - (a) Easy to implement in practice
  - (b) Results are precise.

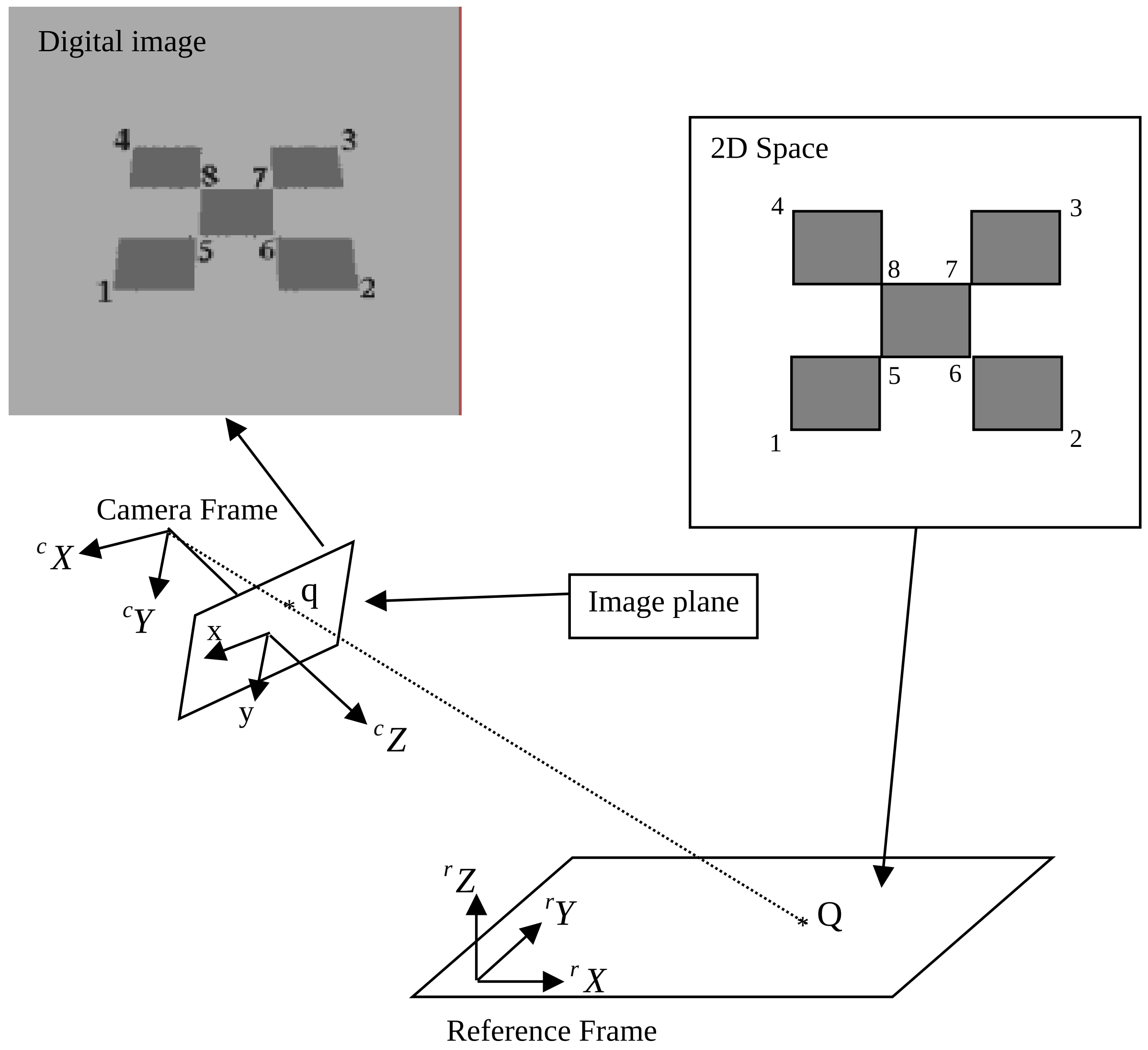


Principle of Calibration in 2D Vision :

Basic idea:

A regular patterns are painted onto the plane of a 2D space.

The real dimensions of the patterns are known. From the digital image, some (at least four) image points are manually detected. Once we know the image coordinates and the real 2D coordinates of these points, the 2D calibration matrix can be computed by a least-square estimation.



Digital image
4
3
8
7
5
6
1
2
2D Space
4
3
8
7
5
6
1
2
Camera Frame
^cX
^cY
q
x
y
^cZ
Image plane
^rZ
^rY
^rX
Q
Reference Frame

Procedure:

Step 1: 2D object coordinates and image coordinates are related by a 2D calibration matrix:

$$\begin{pmatrix} k\bullet{}^{r}X \\ k\bullet{}^{r}Y \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Step 2: Matrix form can be extended into the algebraic form:

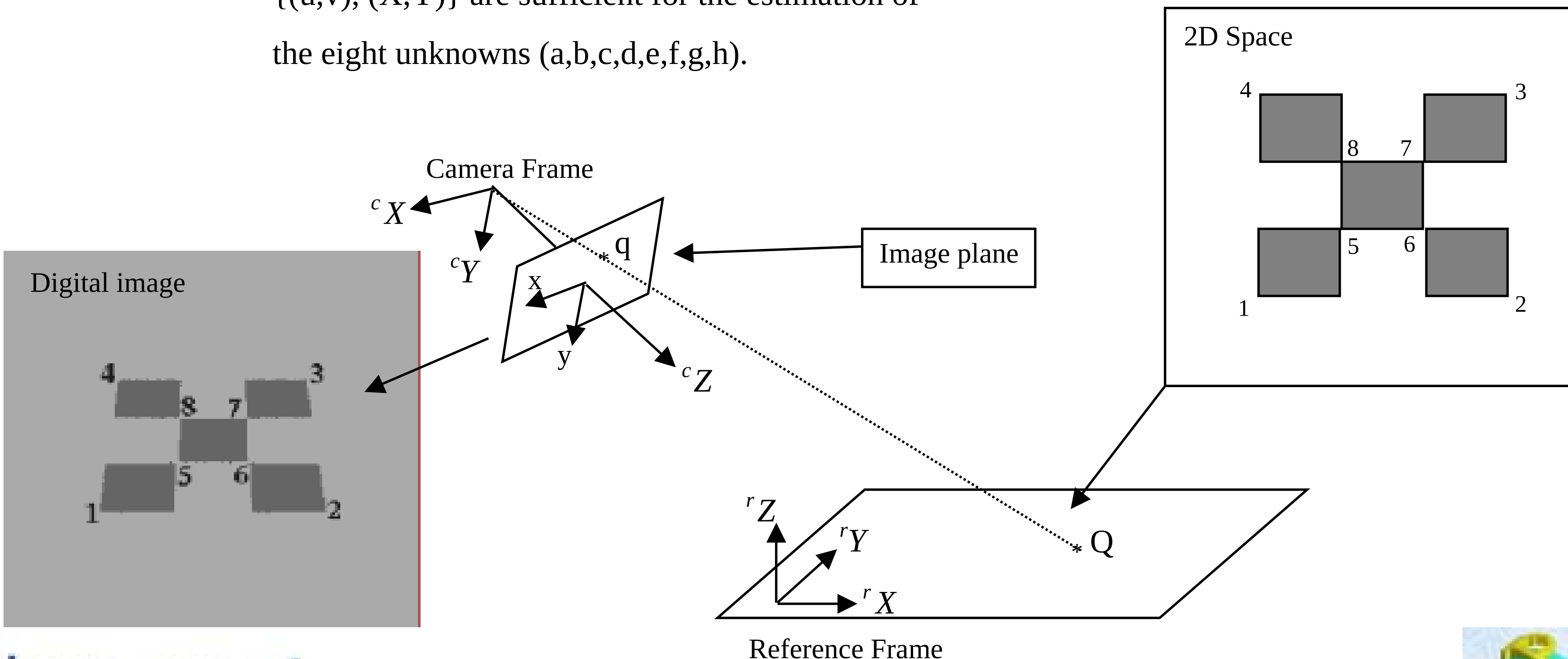
$$\begin{cases} {}^{r}X = \dfrac{u\bullet a + v\bullet b + c}{u\bullet g + v\bullet h + 1} \\ \\ {}^{r}Y = \dfrac{u\bullet d + v\bullet e + f}{u\bullet g + v\bullet h + 1} \end{cases}$$

Step 3: If (u,v) and (X,Y) are known, the above two equations can be written in the following form:

$$\begin{cases} u\bullet a + v\bullet b + c - ({}^{r}X\bullet u)\bullet g - ({}^{r}X\bullet v)\bullet h = {}^{r}X. \\ u\bullet d + v\bullet e + f - ({}^{r}Y\bullet u)\bullet g - ({}^{r}Y\bullet v)\bullet h = {}^{r}Y. \end{cases}$$

Step 4: Given one pair of {(u.v), (X,Y)}, we can have two equations that are linear with respect with the unknown (a,b,c,d,e,f,g,).

Step 5: There are eight unknowns to estimate. We need to have at least eight equations. Therefore, four pairs of {(u,v), (X,Y)} are sufficient for the estimation of the eight unknowns (a,b,c,d,e,f,g,h).



Step 6: If we have "n" pairs of {(ui, vi), (Xi, Yi)} (i=1, 2, …, n),

we can establish "2xn" linear equations:

$$\begin{cases} u_i \bullet a + v_i \bullet b + c - ({}^rX_i \bullet u_i) \bullet g - ({}^rX_i \bullet v_i) \bullet h = {}^rX_i. \\ u_i \bullet d + v_i \bullet e + f - ({}^rY_i \bullet u_i) \bullet g - ({}^rY_i \bullet v_i) \bullet h = {}^rY_i. \end{cases}$$

$$i = 1,2,\ldots,n$$

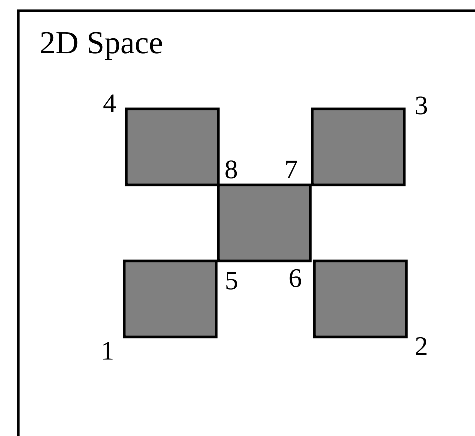
$$A \bullet Z = B$$

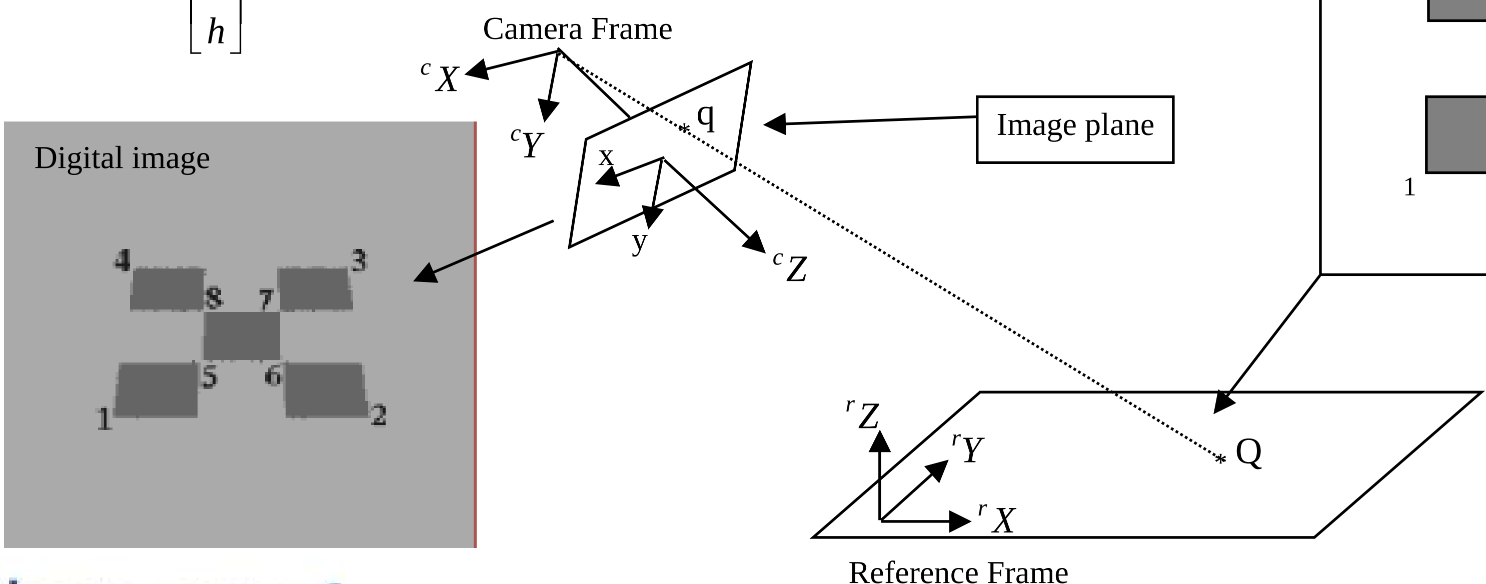
$$Z = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} \quad A = \begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -{}^rX_1 \bullet u_1 & -{}^rX_1 \bullet v_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -{}^rY_1 \bullet u_1 & -{}^rY_1 \bullet v_1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ u_n & v_n & 1 & 0 & 0 & 0 & -{}^rX_n \bullet u_n & -{}^rX_n \bullet v_n \\ 0 & 0 & 0 & u_n & v_n & 1 & -{}^rY_n \bullet u_n & -{}^rY_n \bullet v_n \end{bmatrix} \quad B = \begin{bmatrix} {}^rX_1 \\ {}^rY_1 \\ \ldots \\ {}^rX_n \\ {}^rY_n \end{bmatrix}$$

Step 7: The solution with the least squared error is:

$$Z = (A^t \bullet A)^{-1} \bullet (A^t \bullet B)$$

$$Z = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} \quad A = \begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -{}^rX_1 \bullet u_1 & -{}^rX_1 \bullet v_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -{}^rY_1 \bullet u_1 & -{}^rY_1 \bullet v_1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ u_n & v_n & 1 & 0 & 0 & 0 & -{}^rX_n \bullet u_n & -{}^rX_n \bullet v_n \\ 0 & 0 & 0 & u_n & v_n & 1 & -{}^rY_n \bullet u_n & -{}^rY_n \bullet v_n \end{bmatrix} \quad B = \begin{bmatrix} {}^rX_1 \\ {}^rY_1 \\ \ldots \\ {}^rX_n \\ {}^rY_n \end{bmatrix}$$





Example:

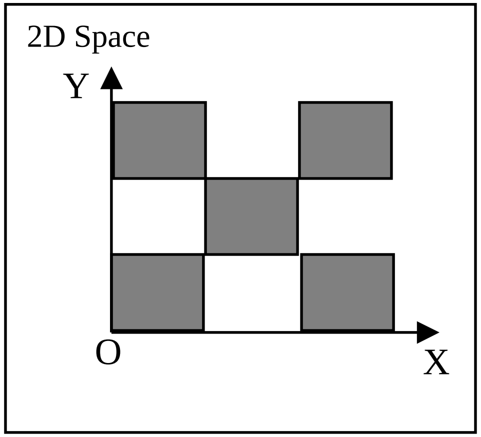
The sizes of the black square in a 2D space are 10.0x10.0 (cm). A CCD camera is fixed at a location above the 2D plane. From the digital image captured by the camera, we select eight image points. The image coordinates of these eight points are:

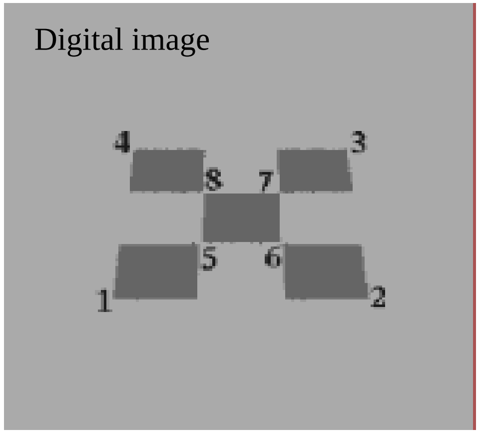
Point 1: (117, 355) Point 2: (396, 356)

Point 3: (372, 175) Point 4: (142, 175)

Point 5: (213, 288) Point 6: (301, 288)

Point 7: (297, 228) Point 8: (215, 227)





What are the coefficients of the 3x3 matrix for this 2D vision system ?

$$Z = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} \quad A = \begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -{}^rX_1 \bullet u_1 & -{}^rX_1 \bullet v_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -{}^rY_1 \bullet u_1 & -{}^rY_1 \bullet v_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ u_n & v_n & 1 & 0 & 0 & 0 & -{}^rX_n \bullet u_n & -{}^rX_n \bullet v_n \\ 0 & 0 & 0 & u_n & v_n & 1 & -{}^rY_n \bullet u_n & -{}^rY_n \bullet v_n \end{bmatrix} \quad B = \begin{bmatrix} {}^rX_1 \\ {}^rY_1 \\ \cdots \\ {}^rX_n \\ {}^rY_n \end{bmatrix}$$

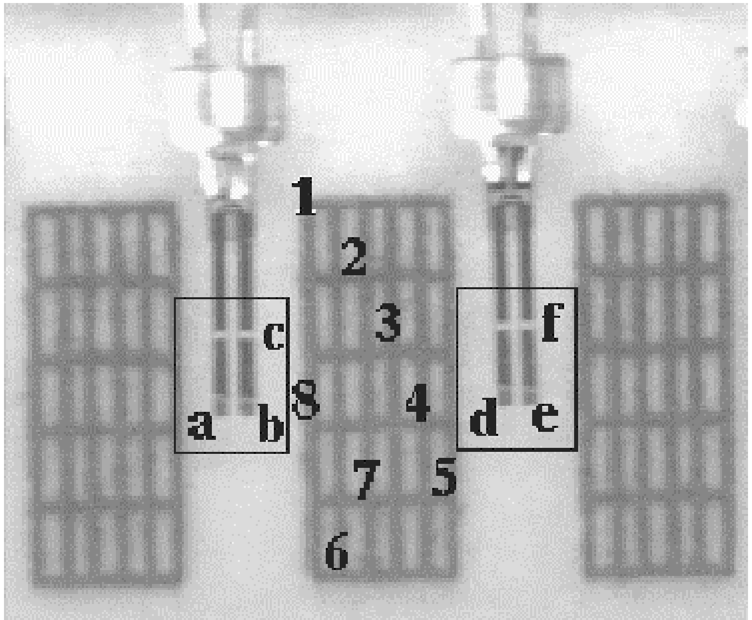
0.164499	0.022269	-27.176669
0.000767	-0.209450	74.306071
0.000018	0.001470	1.0

2D vision only allows to determine the geometry of objects in a 2D space.

Is 2D vision system useful in real applications ?

Example 1: Inspection

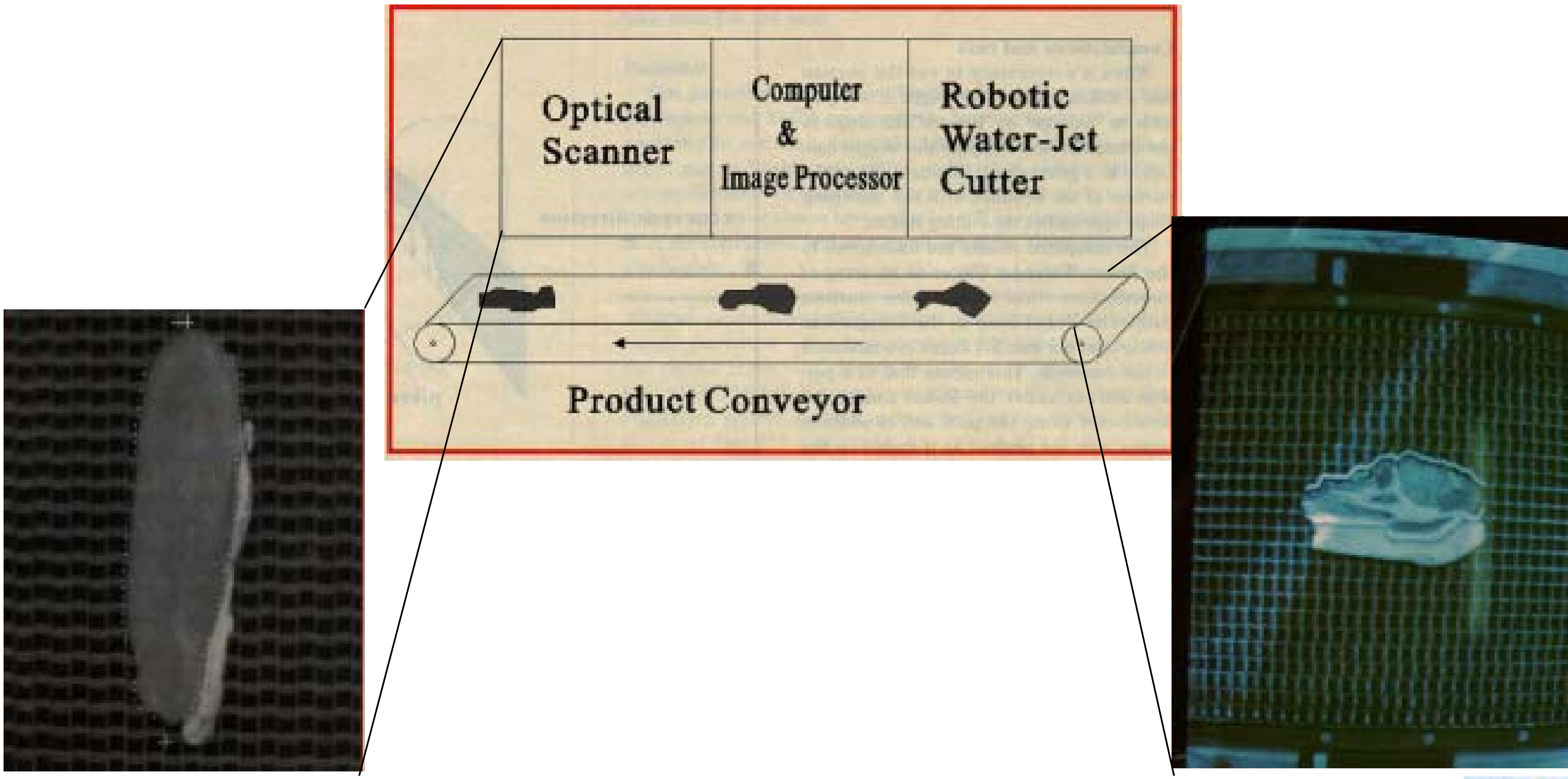
The following photo shows the tuning forks of quartz resonator (necessary device inside every quartz watch). The size of the tuning fork is about 3.0x0.3 (mm). For the quality control, there is a need to check: a) the alignment of the tuning fork and b) the surface dimension of the tuning fork.



Example2: Measurement

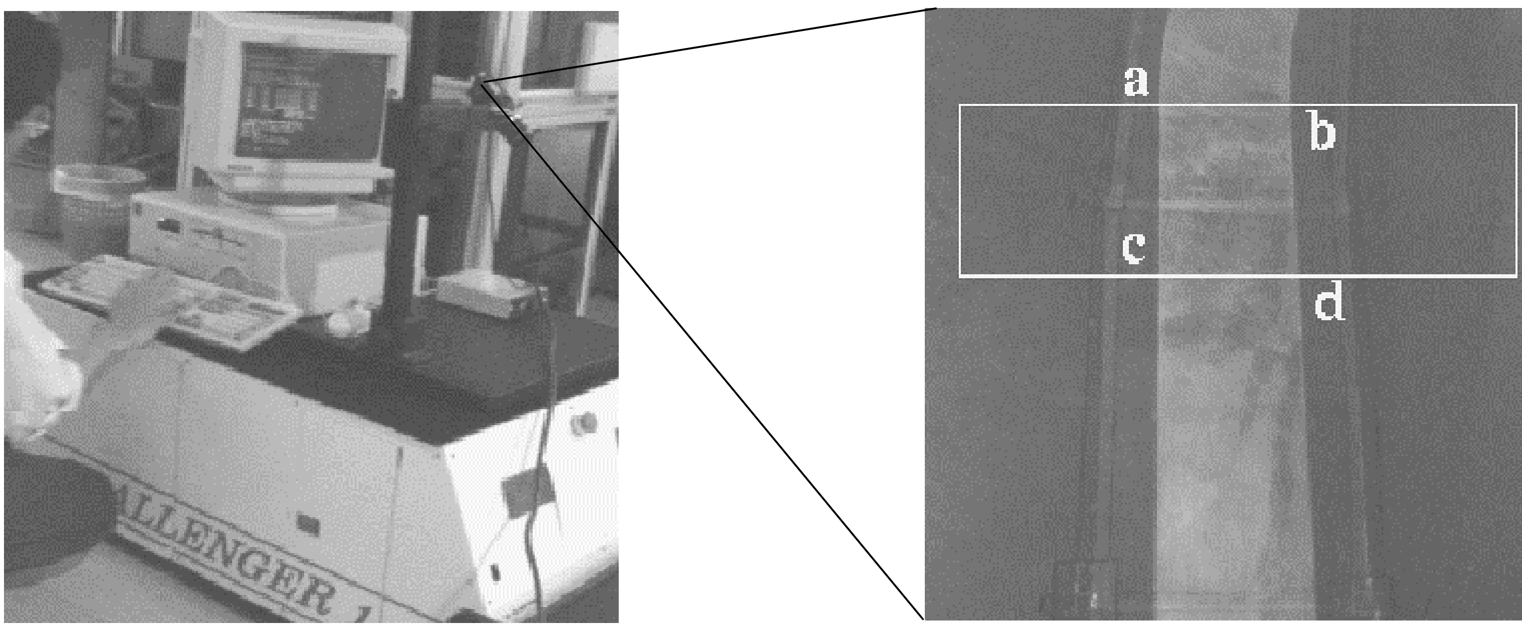
The following photo shows automated system for meat cutting.

The system employs vision system and water-jet cutting machine.



Example3: Vehicle Guidance

The following photos show: a) a 2D vision system mounted on a AGV (Automatic Guided Vehicle) and b) one image seen by the camera. The task for AGV is to follow the land-mark placed on the floor. The floor is assumed to be flat.



SUMMARY

1. 2D object coordinates and image coordinates are related by a 2D calibration matrix:

$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

2. The coefficients of the 2D calibration matrix can be easily determined by a calibration process.

3. One pair of {(u,v), (X,Y)} gives only two linear equations.

4. One needs to have at least four pairs of {(u,v), (X,Y)} to estimate the eight coefficients inside 2D calibration matrix.

5. 2D vision system is useful for industrial applications. It can be used for: a) inspection, b) measurement and c) visual guidance.