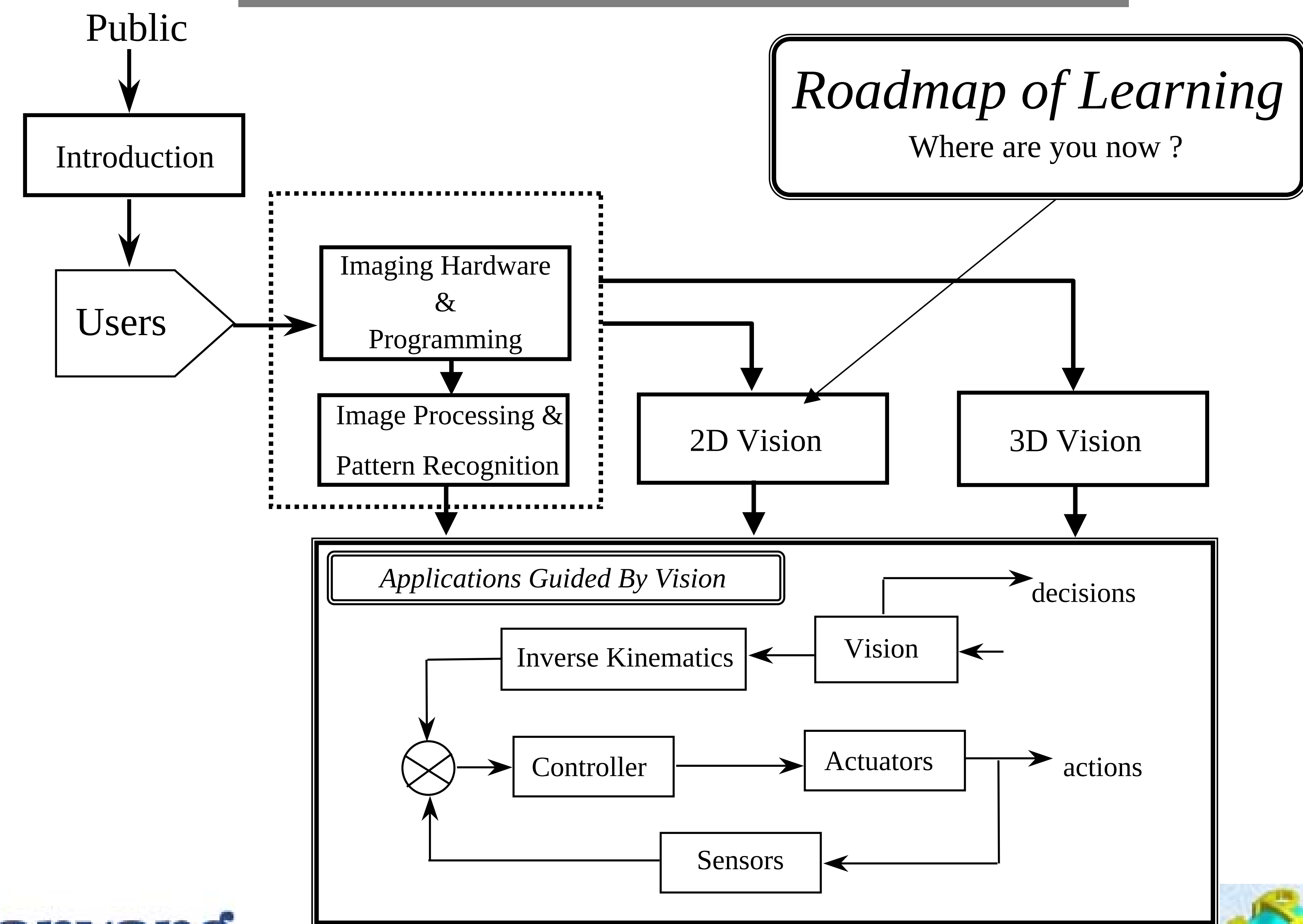
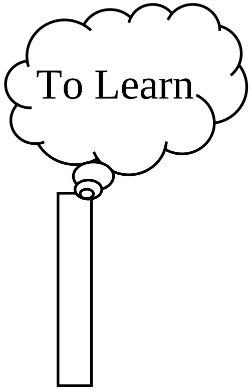


Nanyang Technological University

CONTENT



Chapter 7: Fundamentals of 2D Vision

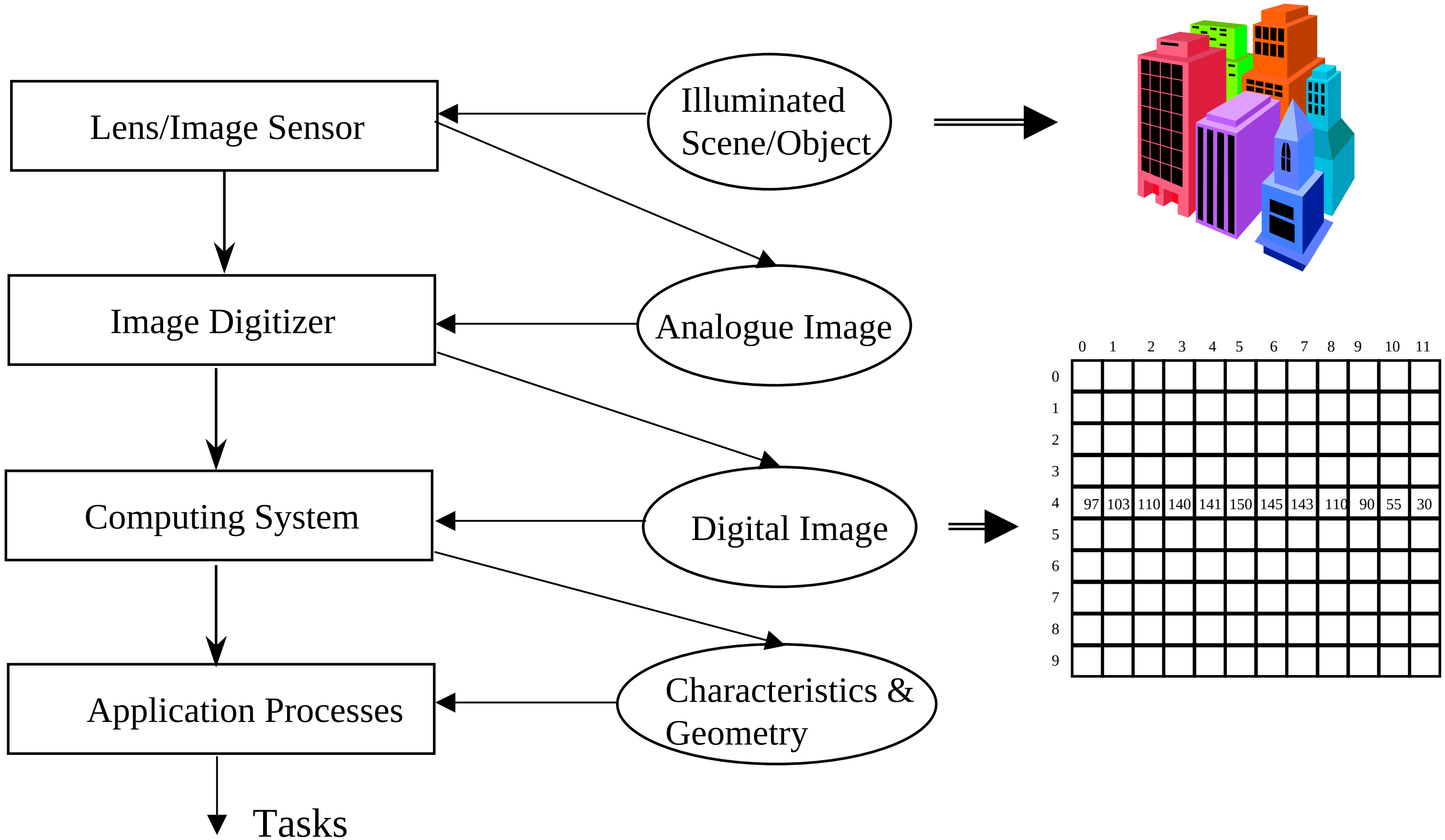
7.1 Geometric Principle of 2D Vision

7.2 Calibration of 2D Vision

7.3 Measurement of 2D Object

What is a machine vision system ? (A Review)

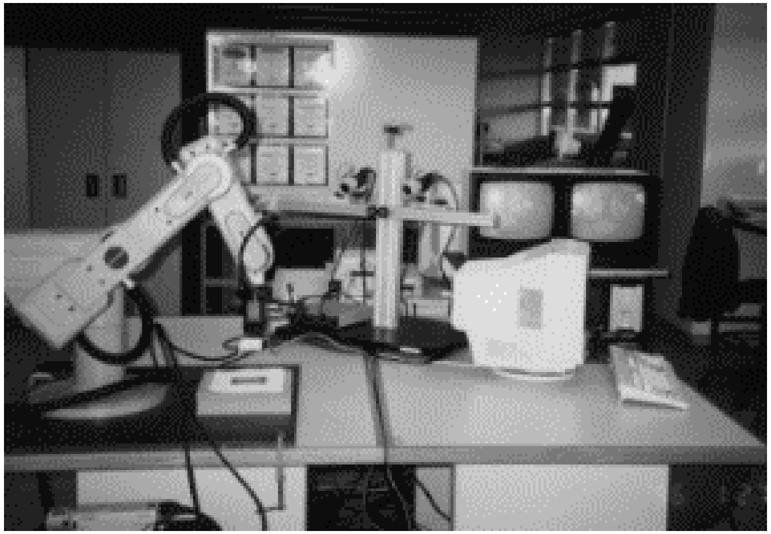
ANSWER:



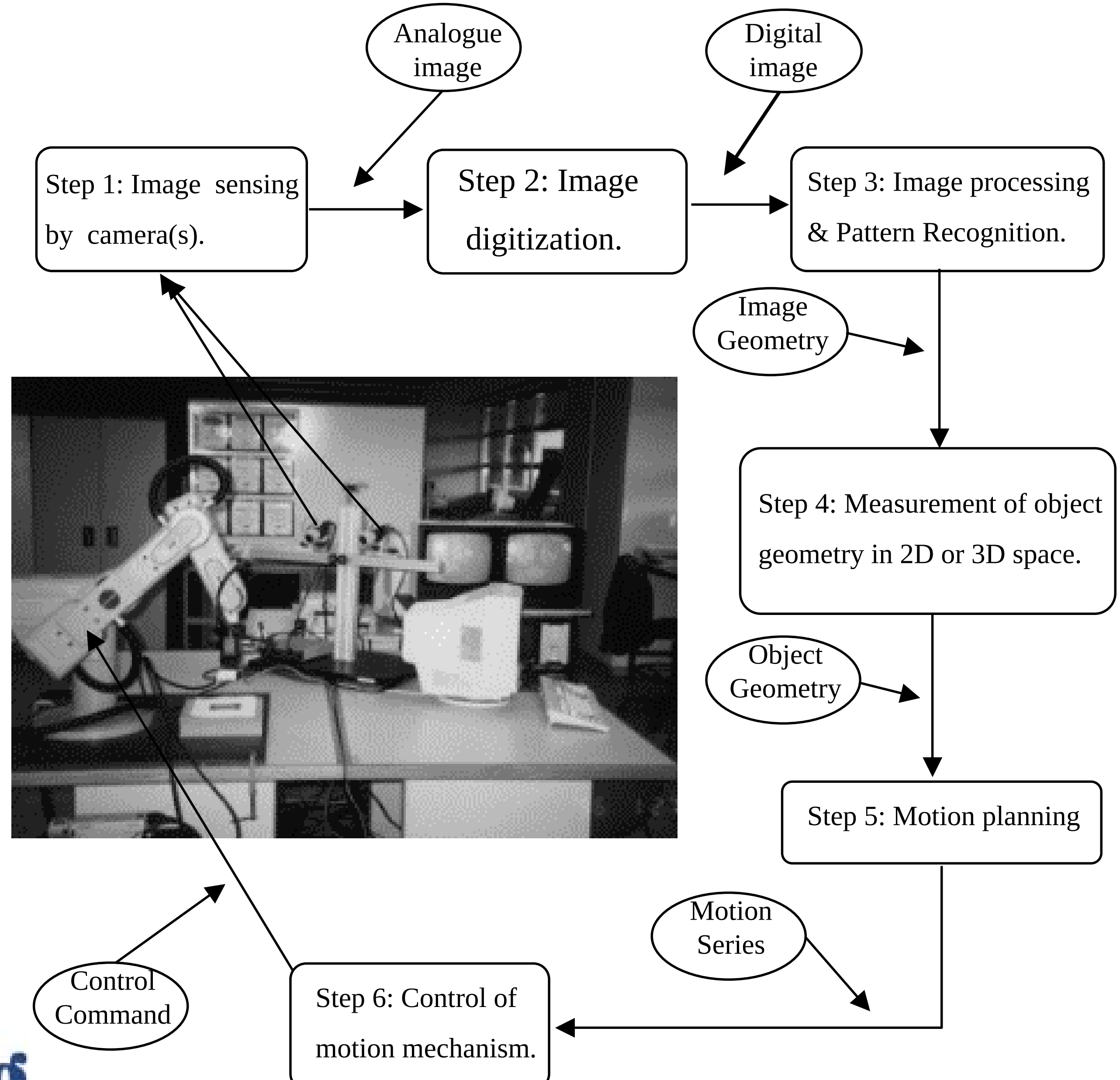
Question:

If we know the geometry of an object inside an image, how to determine the geometry of this object in a 2D space ?

How does a machine vision system work for the task of visual guidance or measurement ?



Answer:

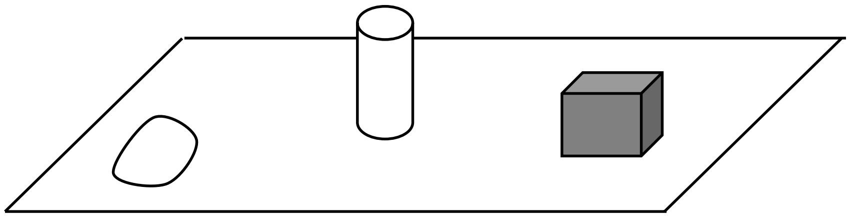

Analogue image
Digital image
Step 1: Image sensing by camera(s).
Step 2: Image digitization.
Step 3: Image processing & Pattern Recognition.
Image Geometry
Step 4: Measurement of object geometry in 2D or 3D space.
Object Geometry
Step 5: Motion planning
Motion Series
Step 6: Control of motion mechanism.
Control Command

How to determine the geometry of an object in a 2D space ?

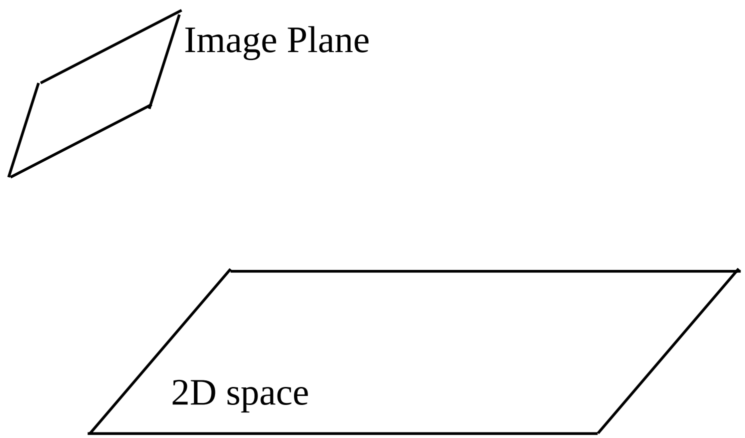
ANSWER:

1. Problem description/analysis:

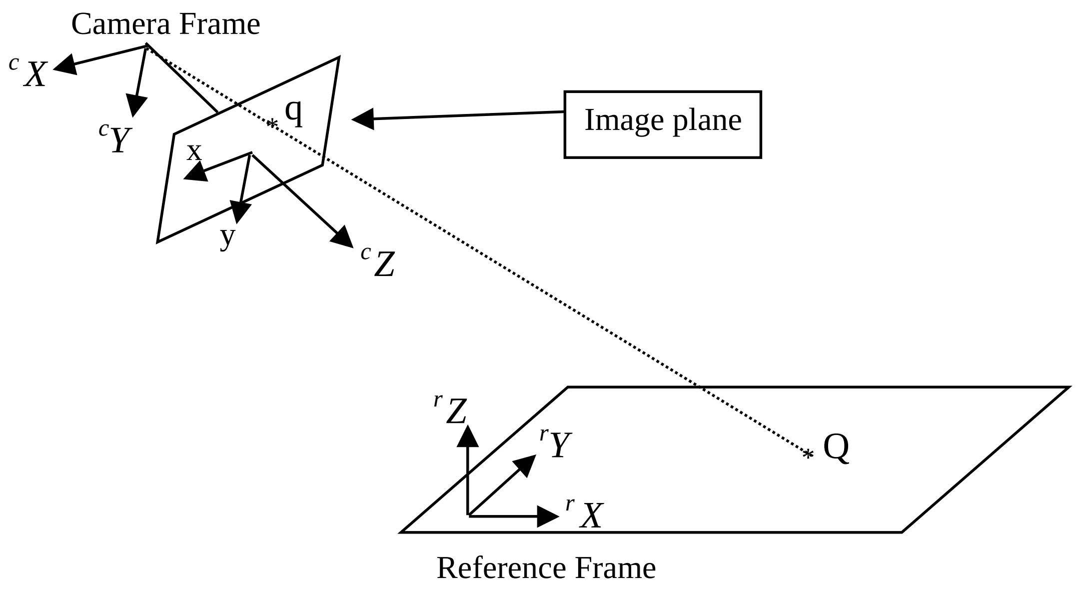
Step 1: In 2D space, all object points are located on a plane.



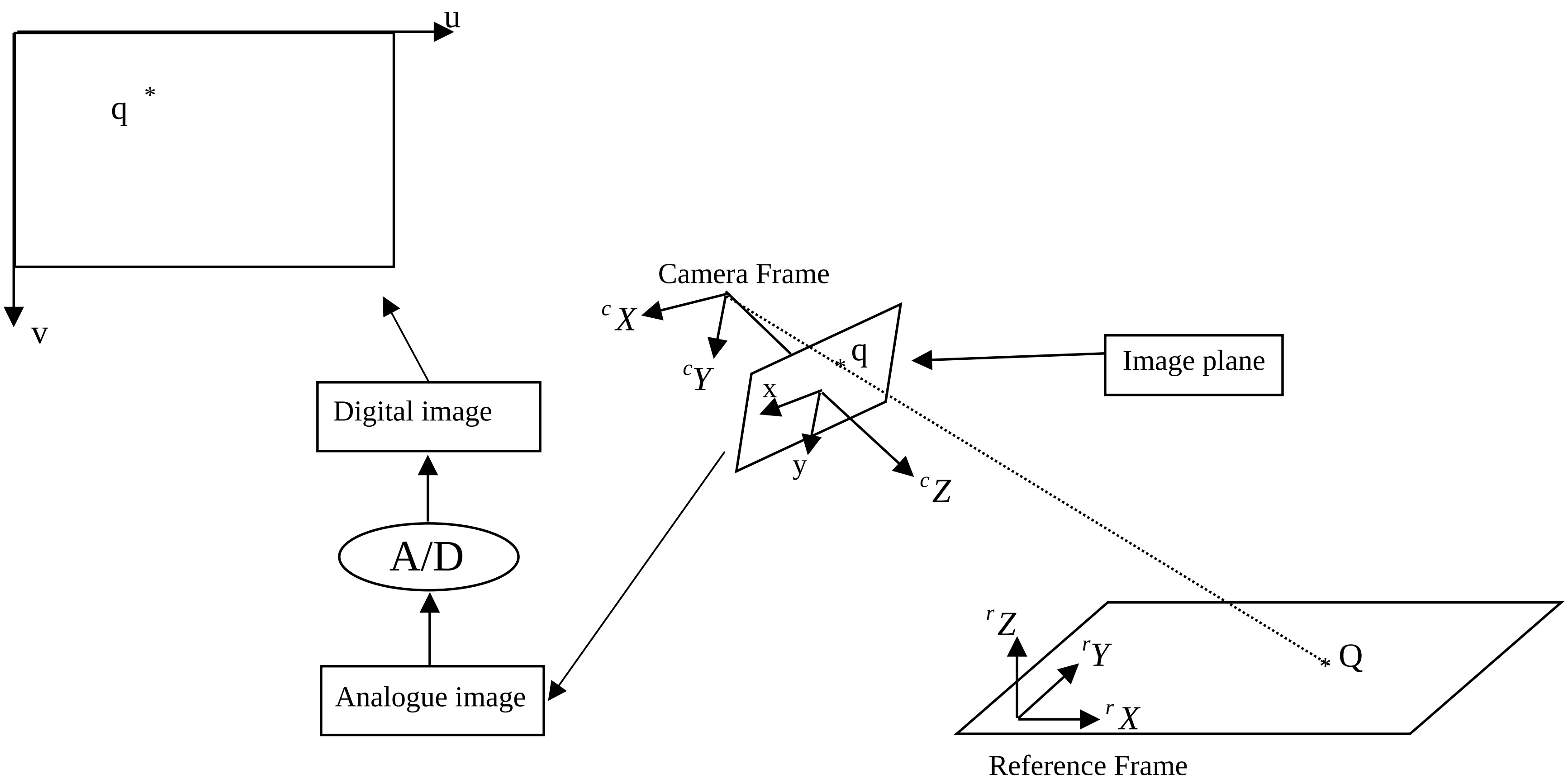
Step 2: The image of an object is captured by a camera whose image plane is not co-planar with the plane of 2D space.



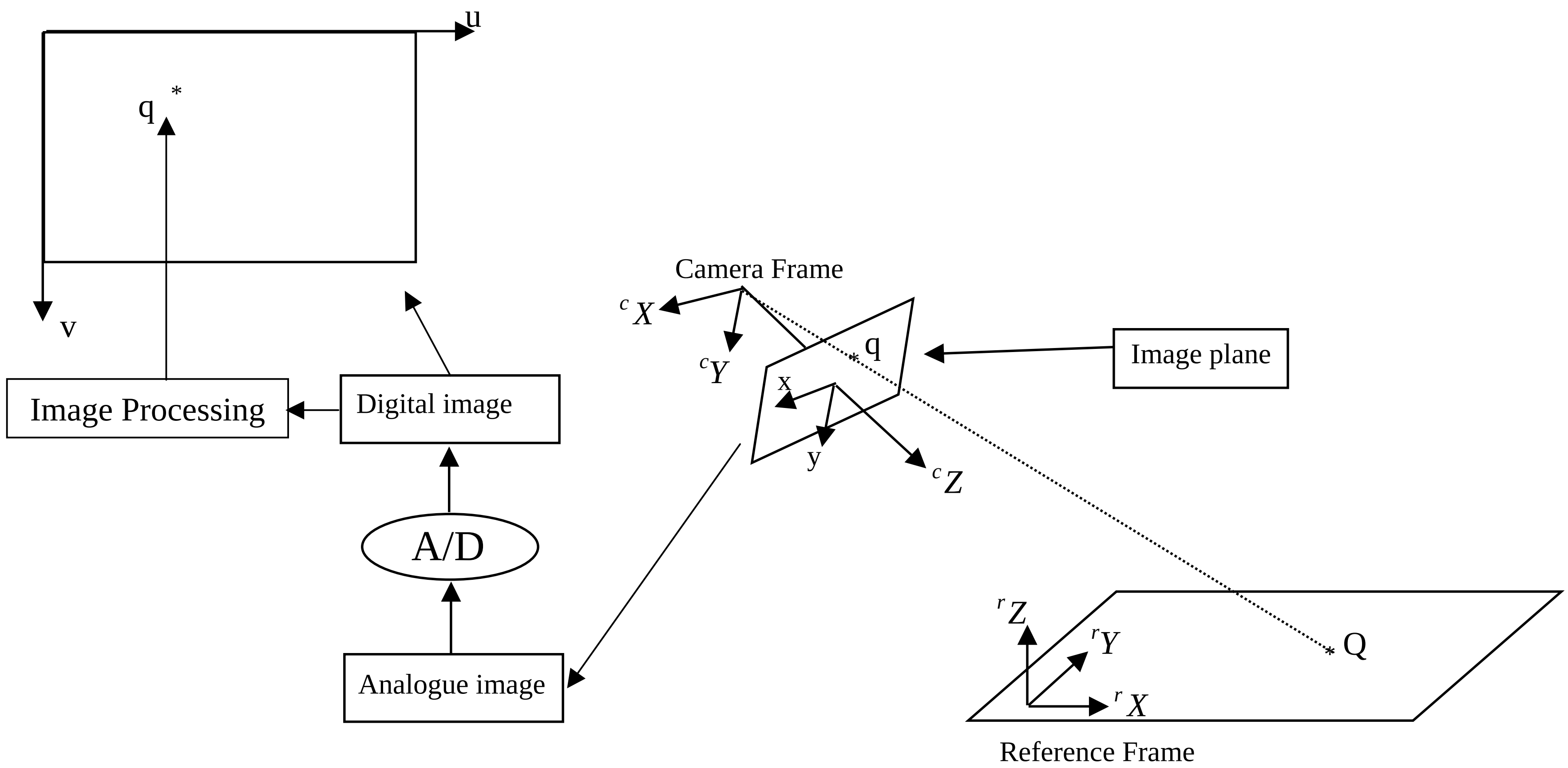
Step 3: An object point is projected onto the image plane according to the geometry of perspective projection.



Step 4: The analogue image formed on the surface of image sensor is converted into the corresponding digital image that is a matrix of pixels.

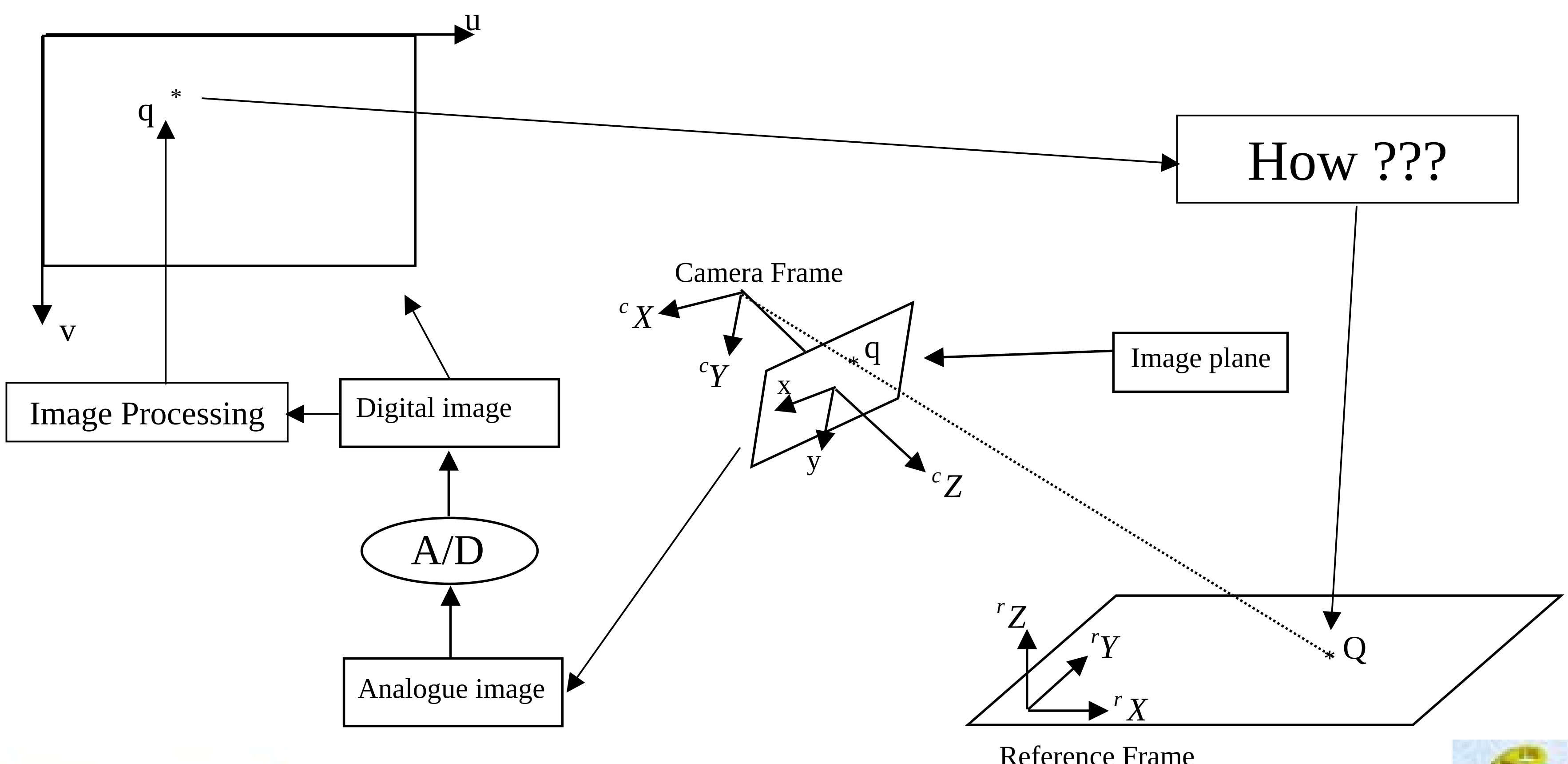


Step 5: By doing image processing and feature extraction, we are able to locate/detect object in a digital image.



Problem Statement:

How to convert the measurement of object geometry in image plane into the measurement of object geometry in 2D space ?



How to determine the geometry of an object in a 2D space ?

ANSWER:

2. Solution:

Input: Image coordinates of a point are : (u, v)

Output: Object coordinates in a 2D space are: $({}^{r}X, {}^{r}Y)$

The solution is:

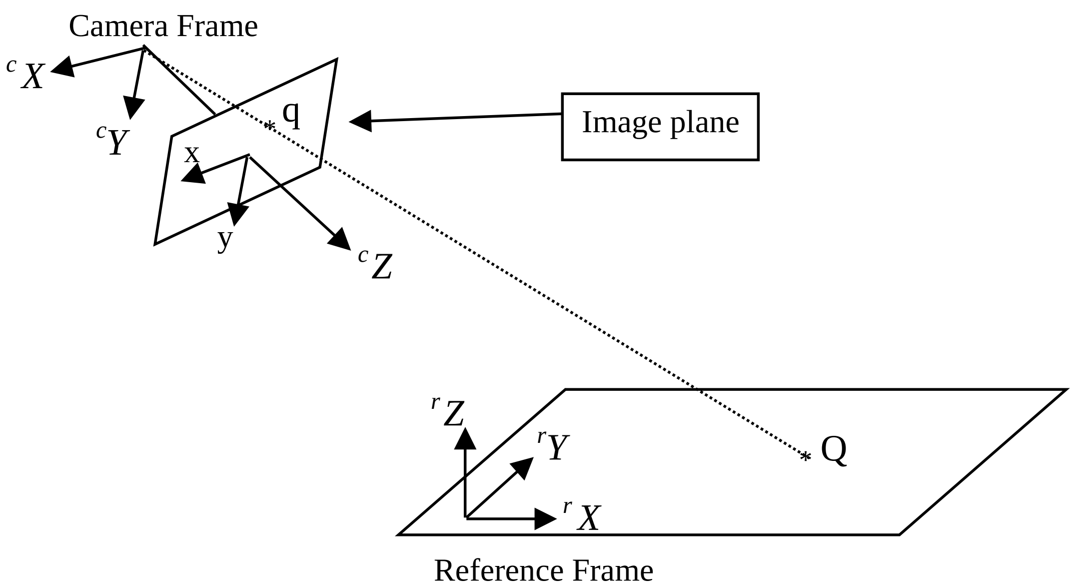
$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \underbrace{\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix}}_{\text{3x3 constant matrix}} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Proof of the solution :

Step 1: Define a reference frame for the 2D space.

Step 2: Define an object frame for the camera.

We call it “camera frame”.

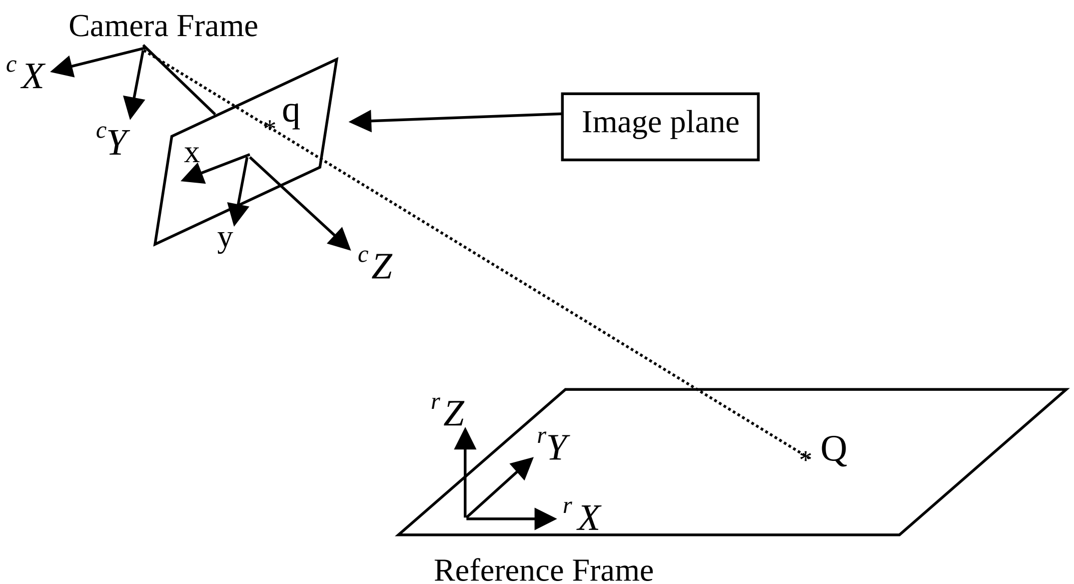


Step 3: If the 3D coordinates of an object point Q in the reference frame are: $({}^rX, {}^rY, {}^rZ)\ ({}^rZ = 0)$, its 3D coordinates in the camera frame will be:

$$\begin{bmatrix} {}^cX \\ {}^cY \\ {}^cZ \\ 1 \end{bmatrix} = {}^cM_r \bullet \begin{bmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{bmatrix}$$

where:

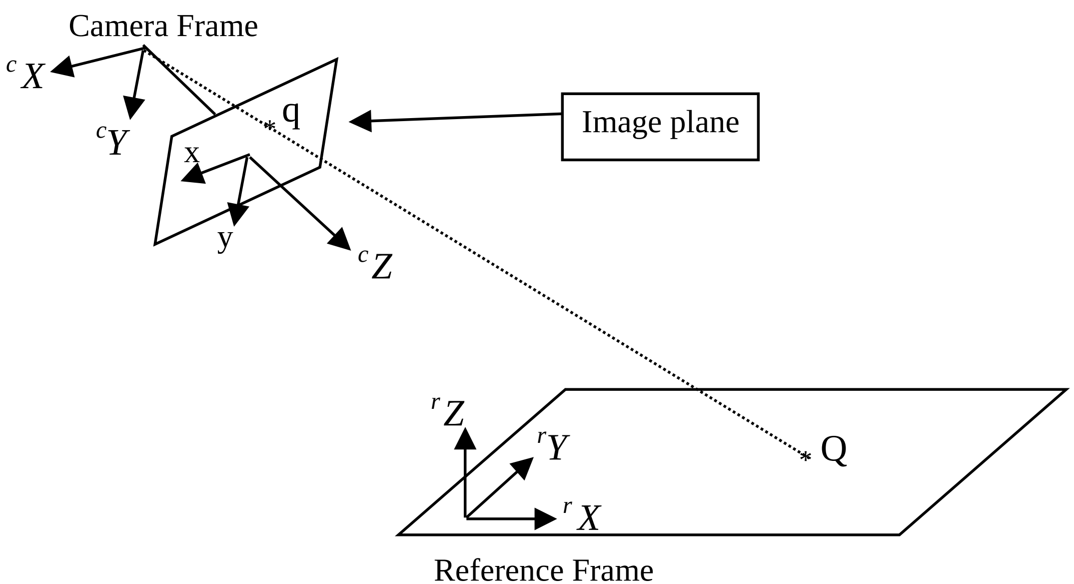
$${}^cM_r = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Step 4: Inside the camera, the point Q is projected into the point q (q is the image of Q).

Step 5: If the real coordinates of the point q are: (x,y), then we have:

$$x = f \bullet \frac{{}^{c}X}{{}^{c}Z} \quad \text{and} \quad y = f \bullet \frac{{}^{c}Y}{{}^{c}Z}$$



$$x = f \bullet \frac{{}^{c}X}{{}^{c}Z} \quad \text{and} \quad y = f \bullet \frac{{}^{c}Y}{{}^{c}Z}$$

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \bullet \begin{pmatrix} {}^{c}X \\ {}^{c}Y \\ {}^{c}Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \bullet ({}^{c}M_{r}) \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

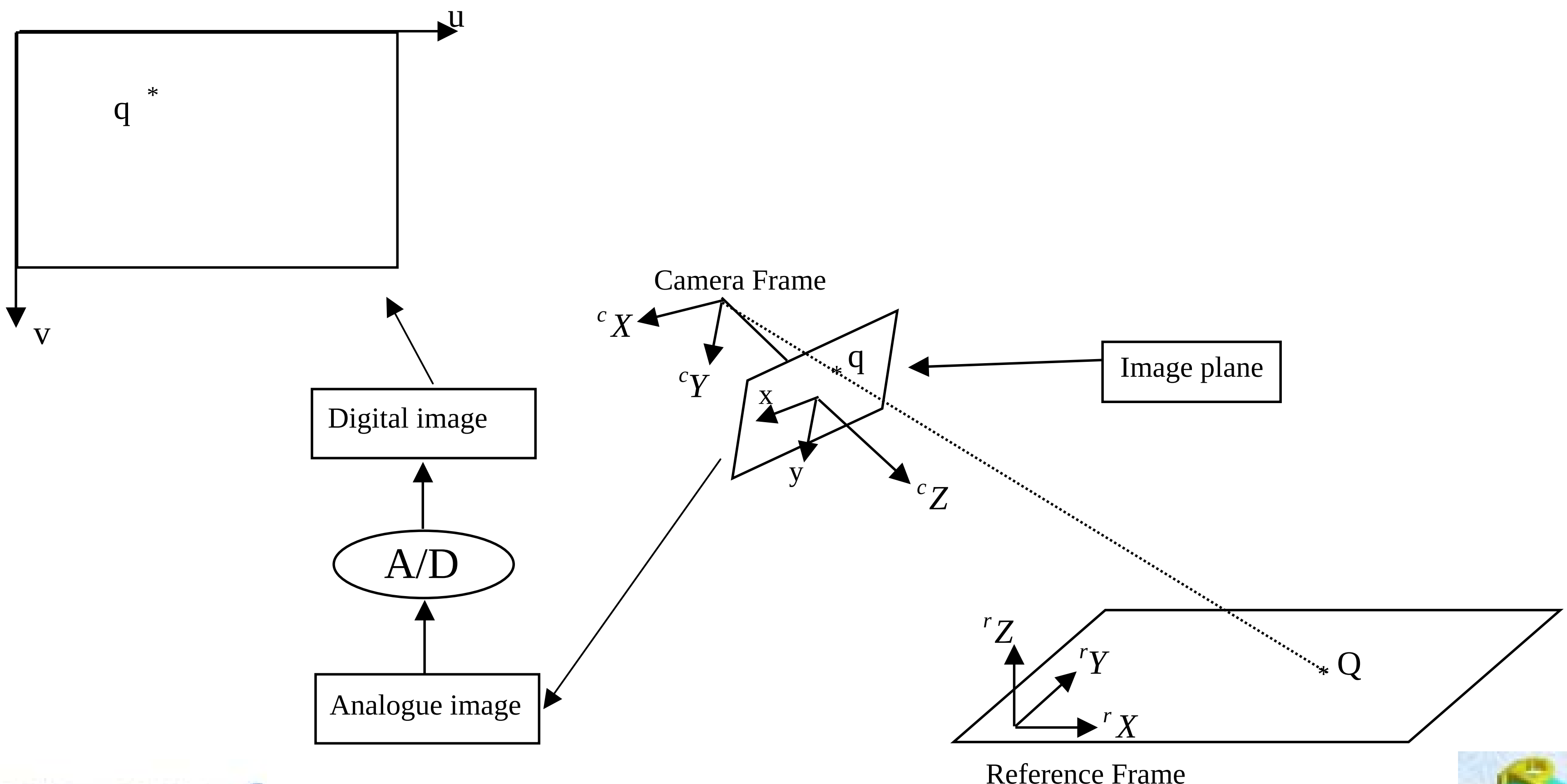
$$\begin{pmatrix} s \bullet x \\ s \bullet y \\ s \end{pmatrix} = A_{3\times 4} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{pmatrix}$$

$$A_{3\times 4} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 6: The analogue image is converted into digital image.

Step 7: If the index coordinates of the point q are (u,v), then we have:

$$u = \frac{x}{D_x} + u_0 \qquad \text{and} \qquad v = \frac{y}{D_y} + v_0$$



$$u = \frac{x}{D_x} + u_0 \quad \text{and} \quad v = \frac{y}{D_y} + v_0$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet A_{3\times4} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = B_{3\times4} \bullet \begin{pmatrix} {}^rX \\ {}^rY \\ {}^rZ \\ 1 \end{pmatrix}$$

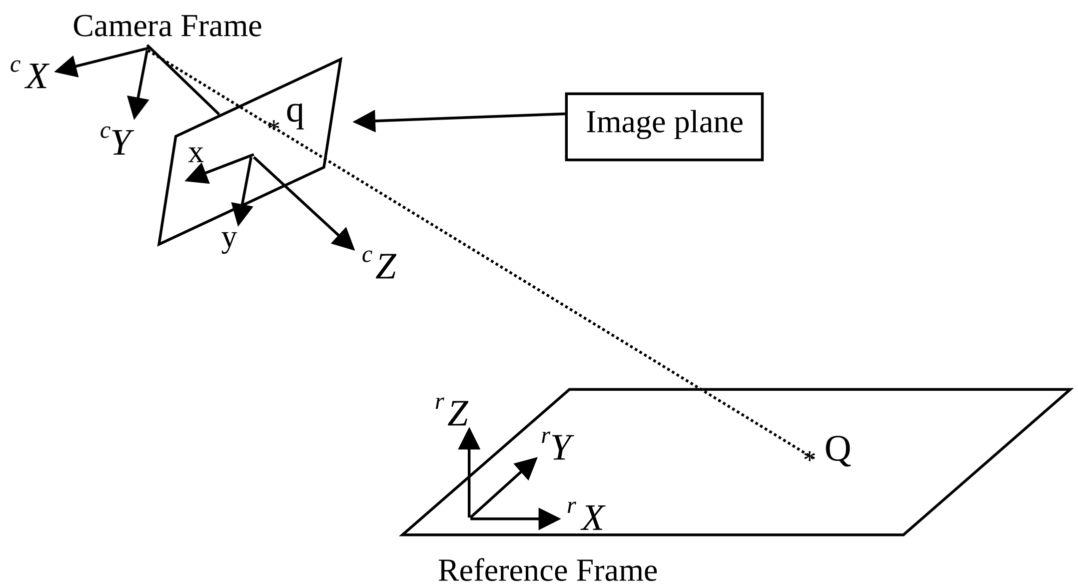
$$B_{3\times4} = \begin{pmatrix} \frac{1}{D_x} & 0 & u_0 \\ 0 & \frac{1}{D_y} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \bullet A_{3\times4}$$

(NOTE: This matrix B is called the “3D Calibration matrix”)

Nanyang Technological University

Step 8: Because we are dealing with object in a 2D space, the Z coordinate is equal to zero (${}^{r}Z = 0$). The replacement of Z with zero value yields:

$$\begin{pmatrix} s \bullet u \\ s \bullet v \\ s \end{pmatrix} = B_{3\times4} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ 0 \\ 1 \end{pmatrix} = C_{3\times3} \bullet \begin{pmatrix} {}^{r}X \\ {}^{r}Y \\ 1 \end{pmatrix}$$



Step 9: $C_{3\times 3}$ is a square and invertible matrix.

If $D_{3\times 3}$ is the inverse of $C_{3\times 3}$, we have:

$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \mathrm{D}_{3\times 3} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

with: k = 1/s, and:

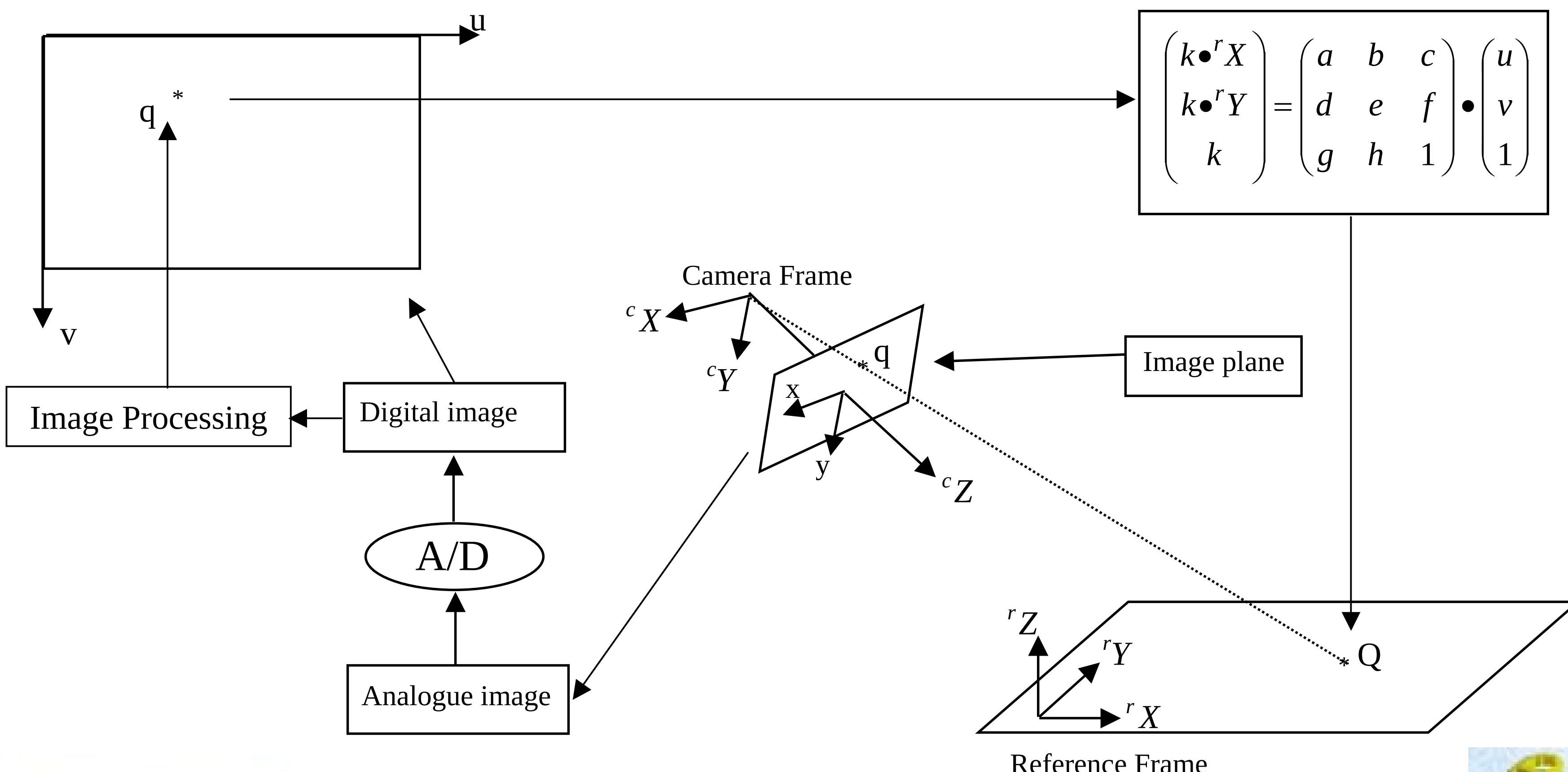
$$D_{3\times 3} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix}$$

(NOTE: This matrix D is called the "2D Calibration matrix")

END OF PROOF

What is the geometric principle of 2D vision ?

ANSWER:



$$\begin{pmatrix} k \bullet {}^rX \\ k \bullet {}^rY \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

SUMMARY

1. The object geometry in a 2D space can be determined from its geometry in a digital image. One camera is sufficient for this purpose.

2. The solution to convert image coordinates into 2D object coordinates is a simple multiplication with a 2D calibration matrix that is a 3x3 matrix:

$$\begin{pmatrix} k \bullet {}^{r}X \\ k \bullet {}^{r}Y \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Quiz 1: *How to determine the coefficients inside the 2D calibration matrix ?*

Quiz 2: *What is the unit of measurement for the image coordinates ?*

What is the unit of measurement for the object coordinates ?

Must they be the same ?