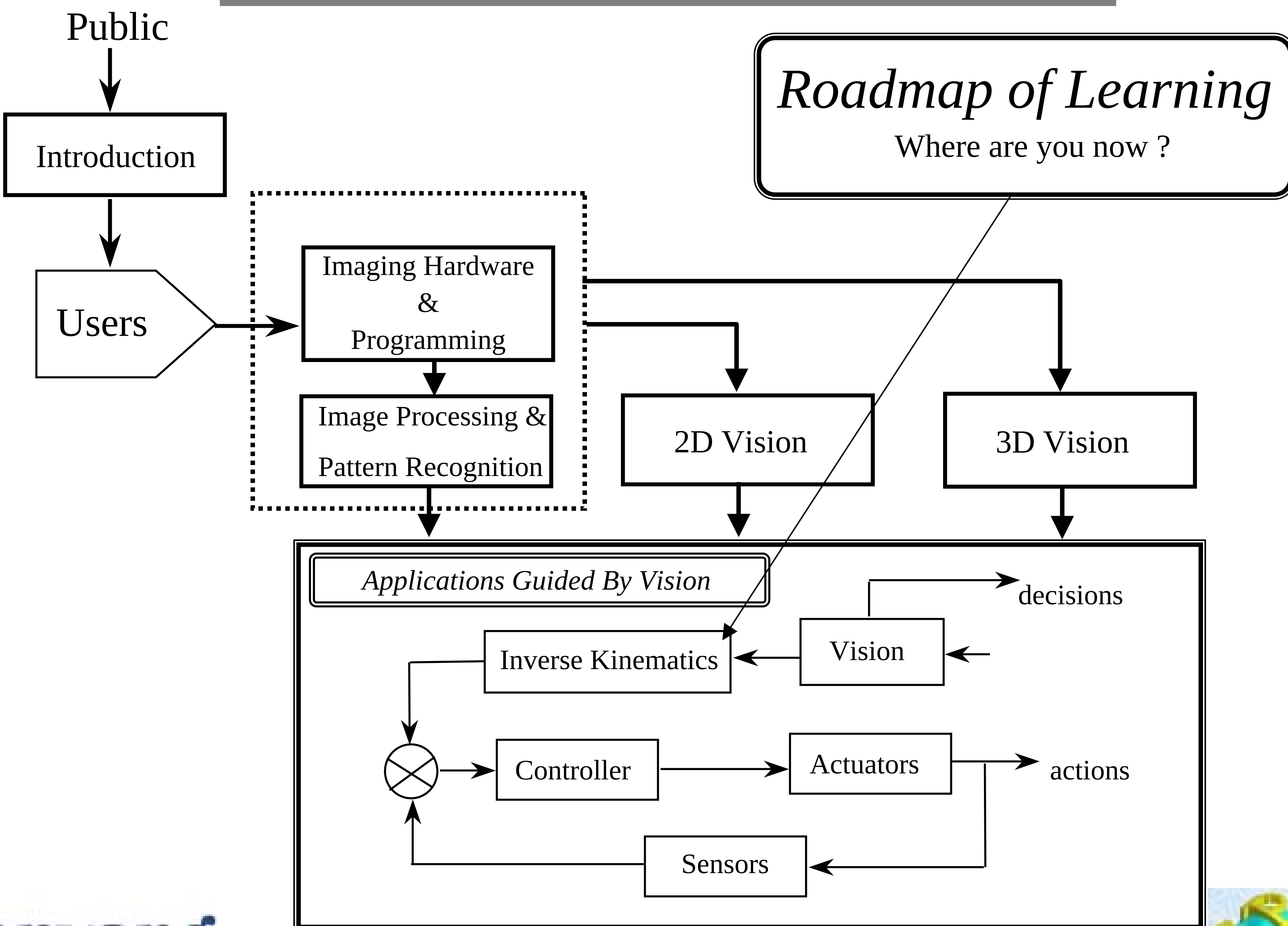

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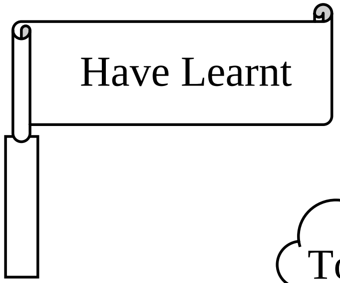
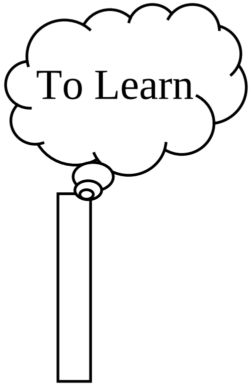
CONTENT

Chapter 6: Representation of Object Location



6.1 Object Location in 2D Space



6.2 Object Location in 3D Space

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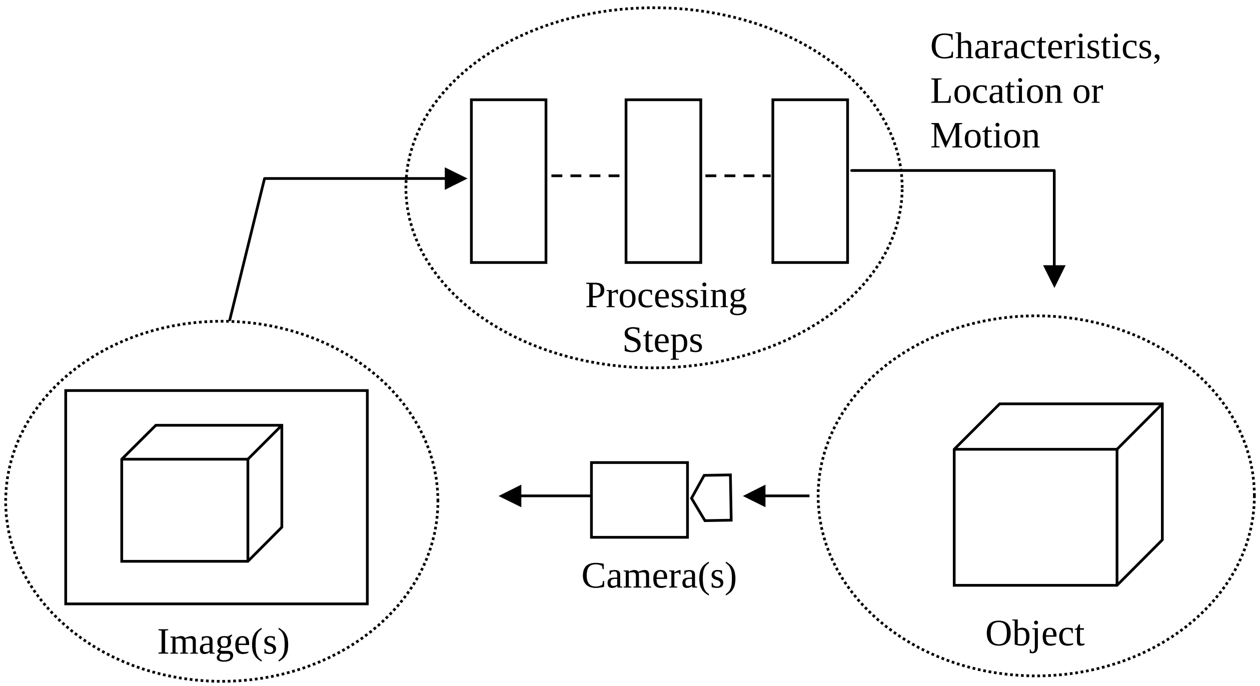
What is machine vision ? (A Review)

ANSWER: It is a process or system that is able to determine object's characteristics, location or motion through the use of images.



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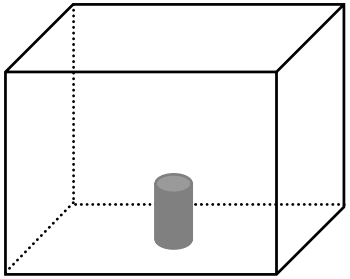
How to define/represent object location in a 3D space ?



ANSWER:

To extend the concepts/principles studied in 2D space into the case of 3D space.

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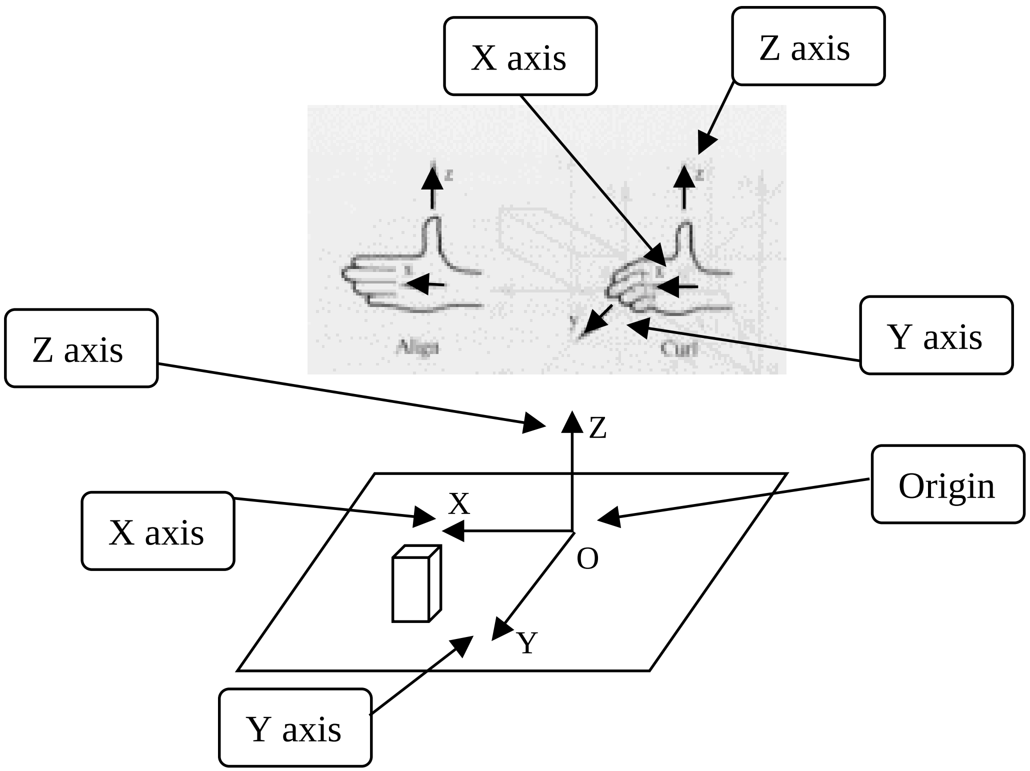
PROCEDURE:

Step 1: Add one more dimension to the case of 2D space according to the “right-hand rule”:



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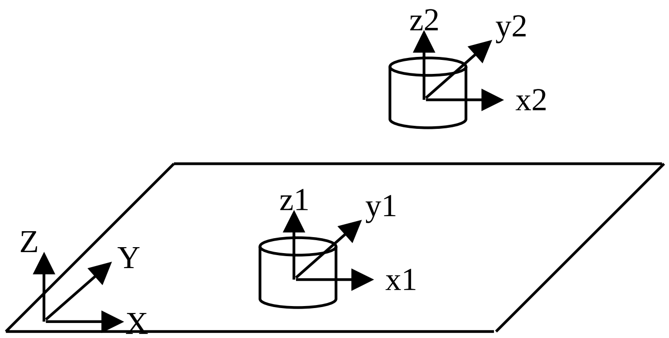
Step 2: Therefore, a point in 3D space is represented by the coordinates (X, Y, Z), and 2D space is a special case of 3D space when considering Z = 0.

Step 3: The concept of "reference frame" and "object frame" is the same in both 2D and 3D space.



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Step 4: The concept of “homogenous coordinates” is the same in both 2D and 3D space.

Step 5: The extension of “motion transformation” from 2D space to 3D space is simply as follows:

$$\begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ {}^{r}Z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} {}^{o}X \\ {}^{o}Y \\ {}^{o}Z \\ 1 \end{bmatrix}$$

The coordinates in the reference frame

Motion transformation matrix in 3D space

The coordinates in object frame.

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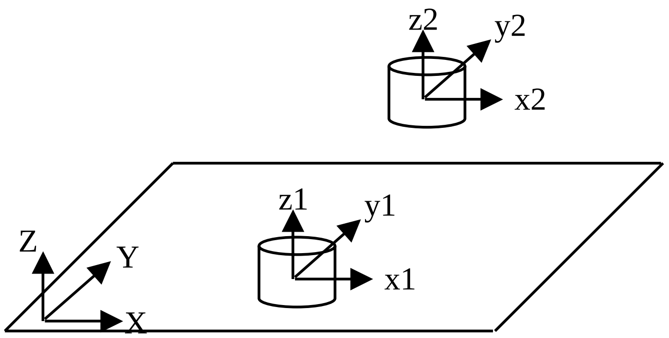
Example 1

To prove that the first column vector of the motion transformation matrix is the unit vector of the object frame's X axis in the reference frame after rotation.



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ANSWER:

1. The unit vector along the object frame's X axis is:

$$ {}^{o}e_x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

2. The above vector in the reference frame is:

$$ {}^{r}e_x = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \\ 1 \end{bmatrix} $$

3. Therefore, the first column is the unit vector of the object frame's X axis in the reference frame after the rotation.

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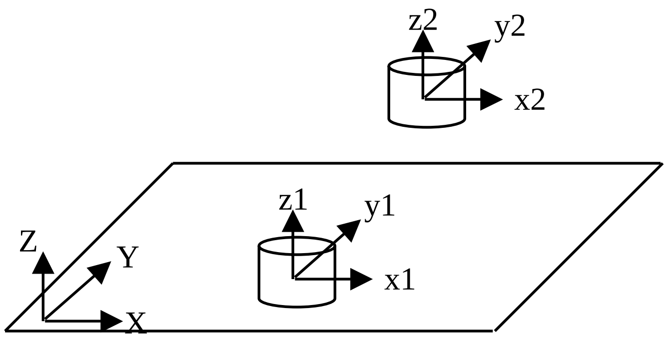
Example 2

To prove that the second column vector of the motion transformation matrix is the unit vector of the object frame's Y axis in the reference frame after rotation.



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ANSWER:

1. The unit vector along the object frame's Y axis is:

$$^{o}e_y = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

2. The above vector in the reference frame is:

$$^{r}e_y = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \\ 1 \end{bmatrix}$$

3. Therefore, the second column is the unit vector of the object frame's Y axis in the reference frame after the rotation.

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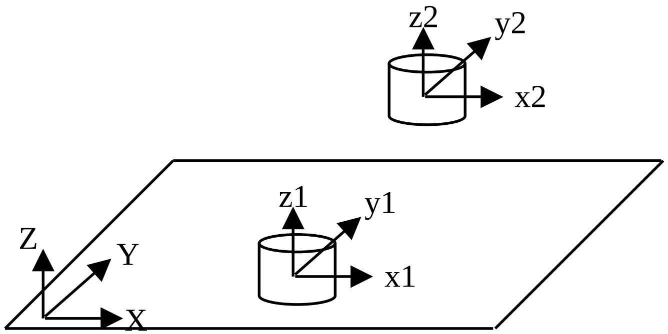
Example 3

To prove that the third column vector of the motion transformation matrix is the unit vector of the object frame's Z axis in the reference frame after rotation.



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ANSWER:

1. The unit vector along the object frame's Z axis is:

$$^{o}e_z = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

2. The above vector in the reference frame is:

$$^{r}e_z = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \\ 1 \end{bmatrix}$$

3. Therefore, the third column is the unit vector of the object frame's Z axis in the reference frame after the rotation.

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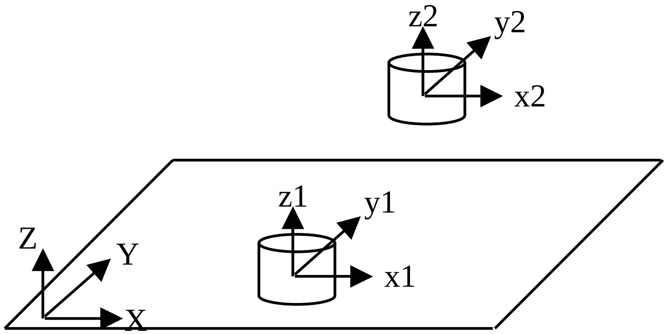
Example 4

What is the motion transformation matrix of a pure translation along X axis by the object frame with respect to the reference frame ?

ANSWER:

$$ {}^{r}M_{o} = Trans(t_x) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



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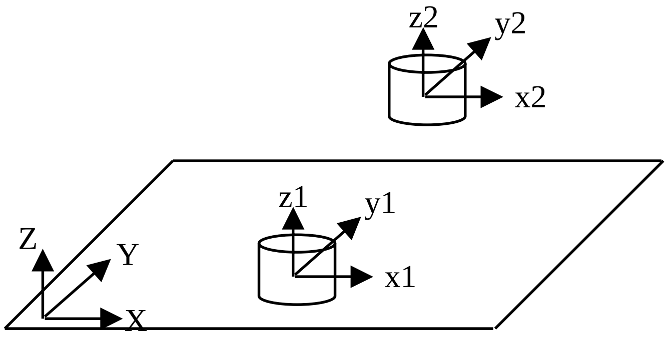
Example 5

What is the motion transformation matrix of a pure translation along Z axis by the object frame with respect to the reference frame ?

ANSWER:

$$ {}^{r}M_{o} = Trans(t_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



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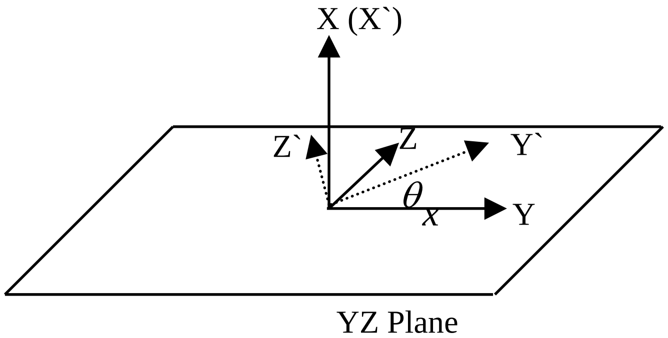
Example 6

What is the motion transformation matrix of a pure rotation around the X axis of the reference frame by the object frame ?



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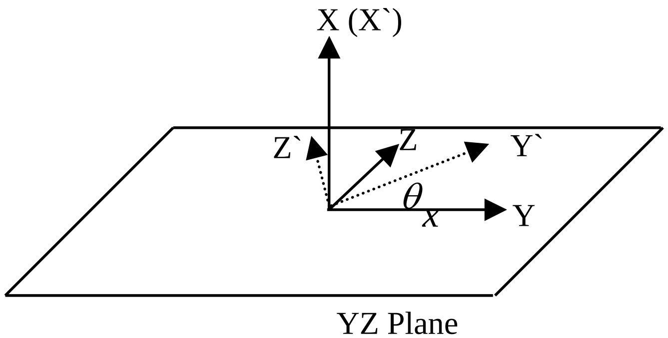
ANSWER:

1. The rotation is around the X axis of the reference frame. The X coordinate does not change after the rotation.

2. The geometric illustration of rotation in the YZ plane is as follows:



3. The motion transformation matrix is:

$$ {}^{r}M_{o} = Rot(\theta_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) & 0 \\ 0 & \sin(\theta_x) & \cos(\theta_x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Quiz: *Do you know how to quickly verify the result ?*

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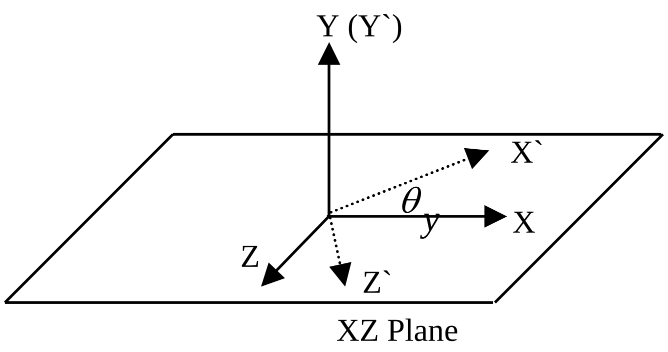
Example 7

What is the motion transformation matrix of a pure rotation around the Y axis of the reference frame by the object frame ?



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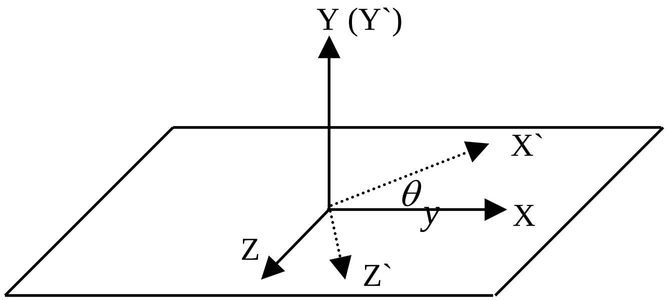
ANSWER:

1. The rotation is around the Y axis of the reference frame. The Y coordinate does not change after the rotation.

2. The geometric illustration of rotation in the XZ plane is as follows:



3. The motion transformation matrix is:

$$^{r}M_{o} = Rot(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Quiz: *Do you know how to quickly verify the result ?*

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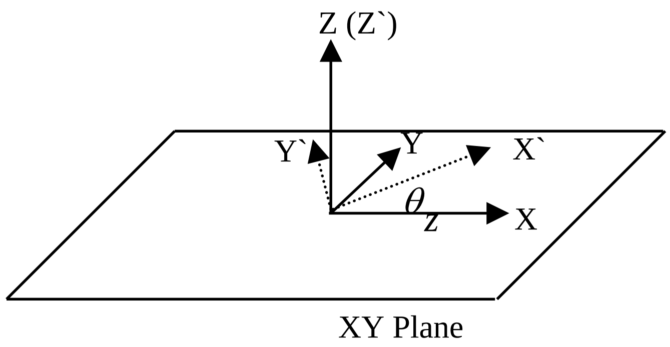
Example 8

What is the motion transformation matrix of a pure rotation around the Z axis of the reference frame by the object frame ?



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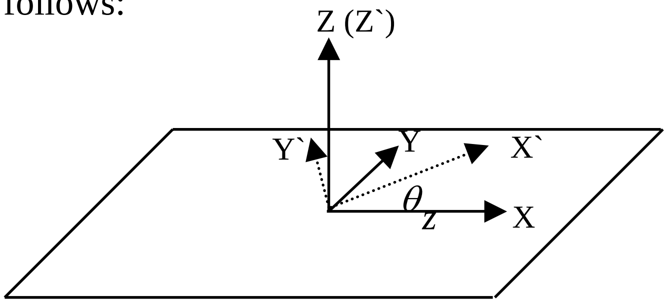
ANSWER:

1. The rotation is around the Z axis of the reference frame. The Z coordinate does not change after the rotation.

2. The geometric illustration of rotation in the XY plane is as follows:



3. The motion transformation matrix is:

$$ {}^{r}M_{o} = Rot(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Quiz: *Do you know how to quickly verify the result ?*

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SUMMARY

1. In 3D space, a reference frame is needed in order to describe object location.

2. An object can have its own coordinate system that is called “object frame”.

3. The relationship between the reference frame and an object frame in 3D space is composed of rotation and translation:

$$
{}^{r}M_{o} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= Trans(t_z)Trans(t_y)Trans(t_z)Rot(\theta_z)Rot(\theta_y)Rot(\theta_x)
$$

$$
{}^{r}P = {}^{r}M_{o} \bullet {}^{o}P
$$

4. The initial position of an object frame is always assumed to be at the origin of the reference frame.

5. Any point is attached to either the “reference frame” or a “object frame”. Hence, motion of a point is equivalent to the motion of the corresponding frame of the point.

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