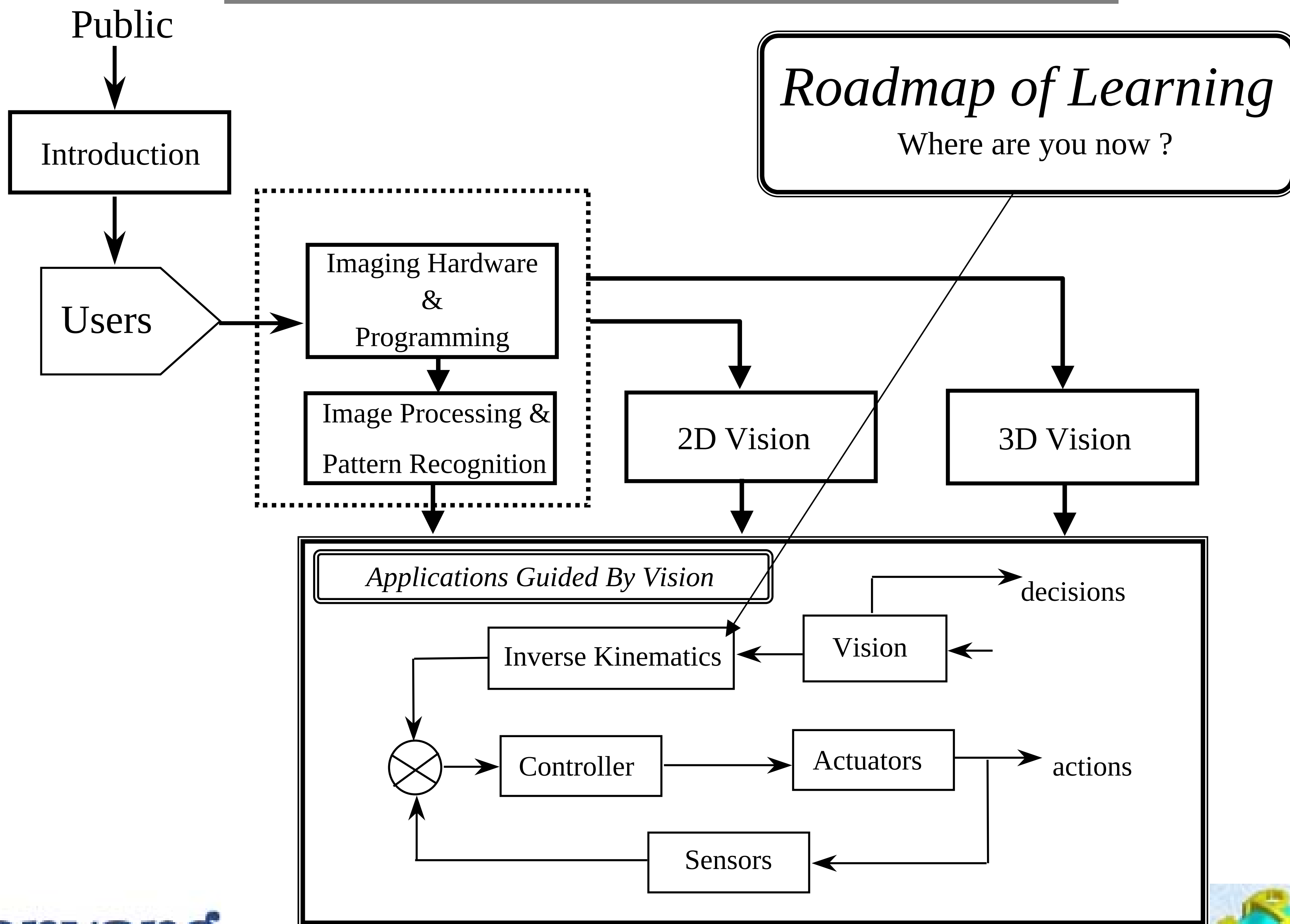

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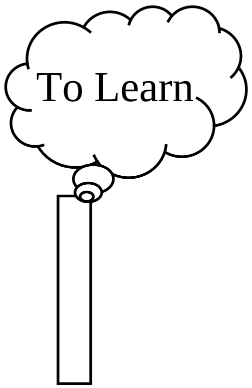
CONTENT

Chapter 6: Representation of Object Location



6.1 Object Location in 2D Space

6.2 Object Location in 3D Space

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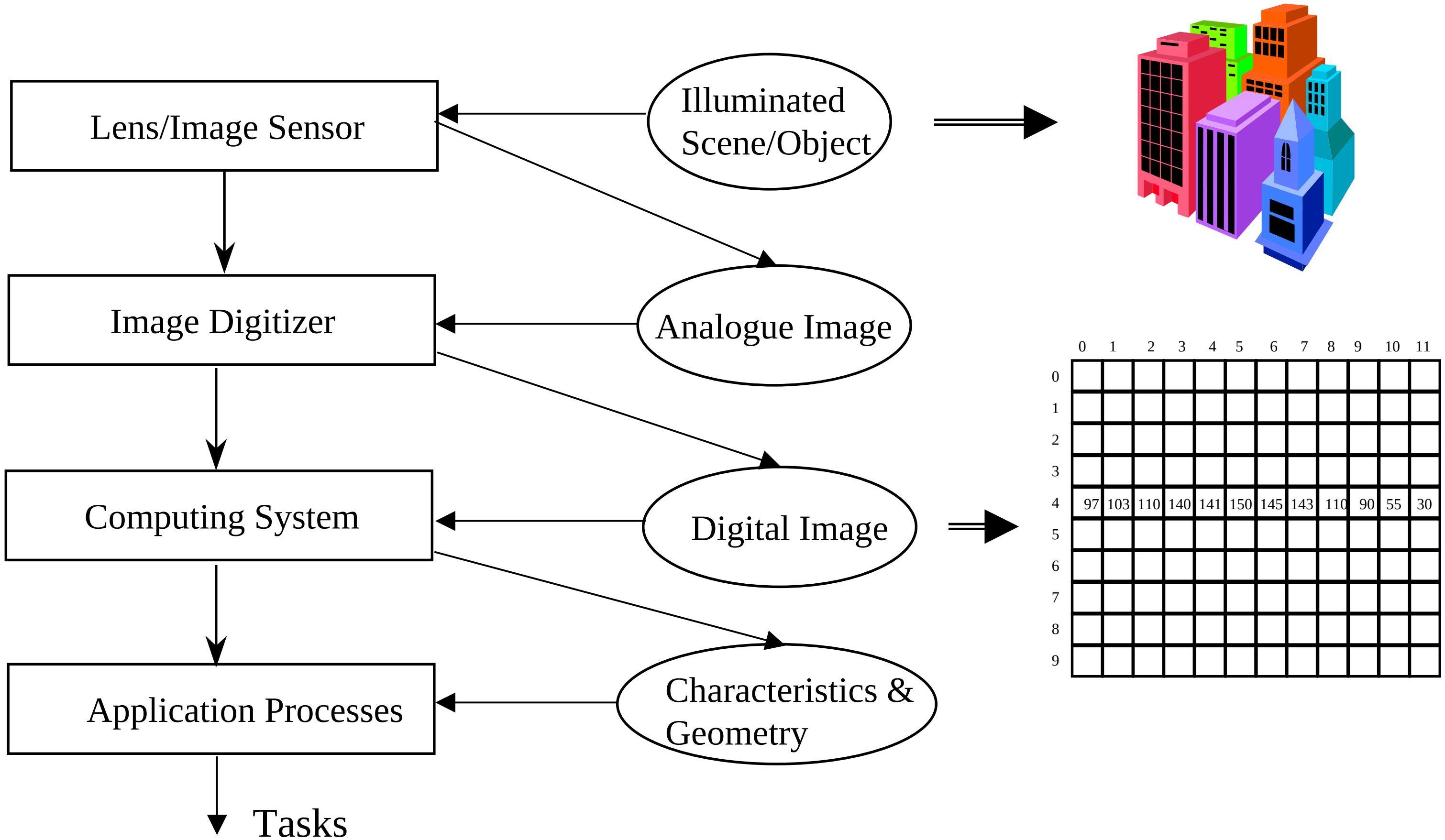
What is a machine vision system ? (A Review)

ANSWER:



Question:

How to represent object’s location/motion in a 2D space ?

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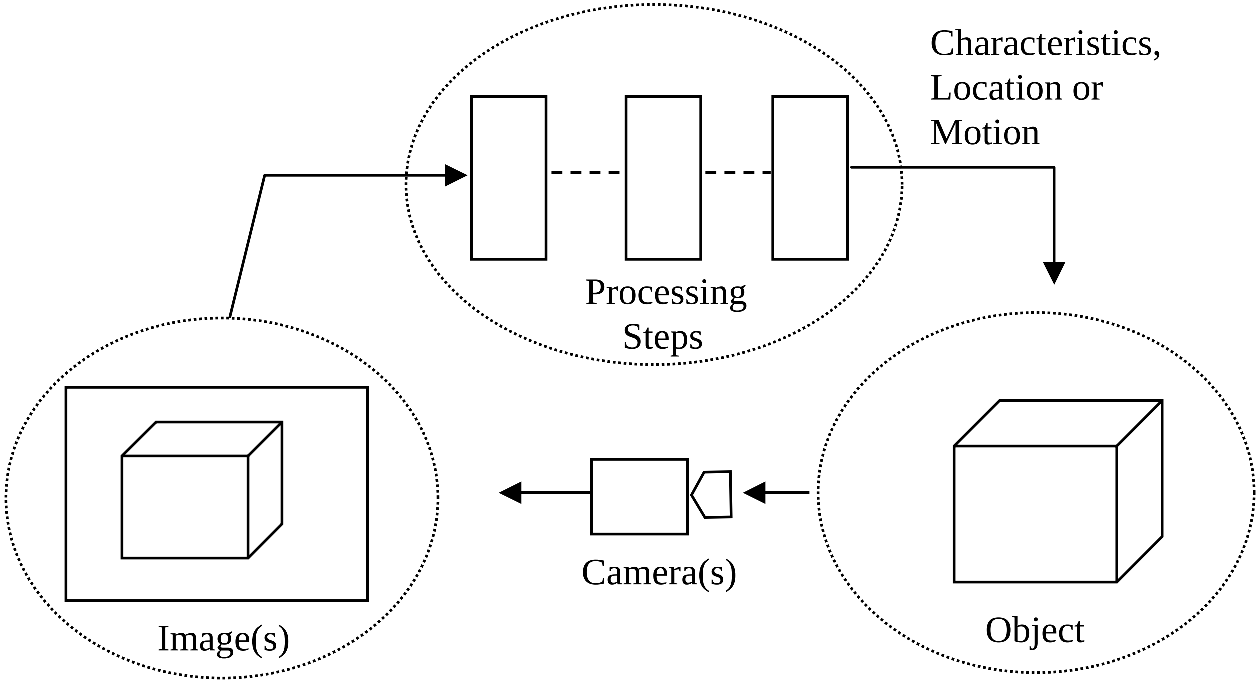
What is machine vision ?

ANSWER: It is a process or system that is able to determine object’s characteristics, location or motion through the use of images.



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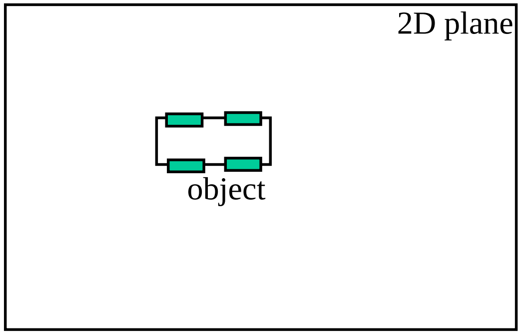
How to define/represent object location in a 2D space ?

Question 1:

How to tell the location of an object in a 2D space ?



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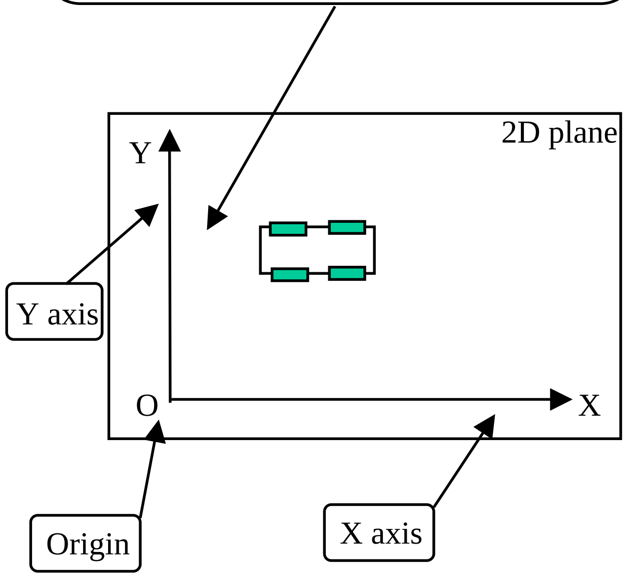
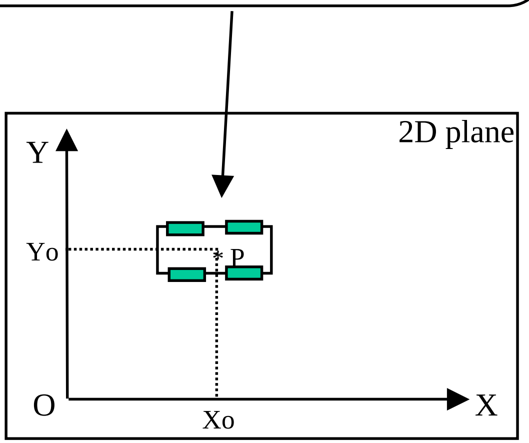
ANSWER:

Step 1: We need to define a reference that is a “coordinate system”. This coordinate system OXY is called “reference frame”.



Step 2: Then, one can measure the position of a point P in terms of its X coordinate Xo and its Y coordinate Yo. Its location is at (Xo, Yo).



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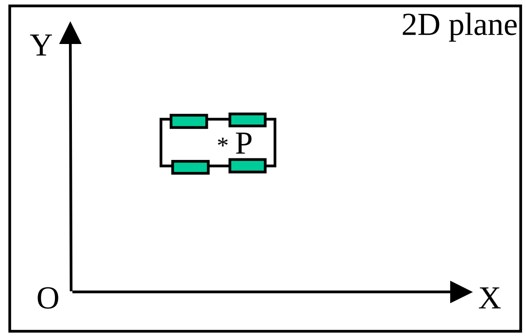
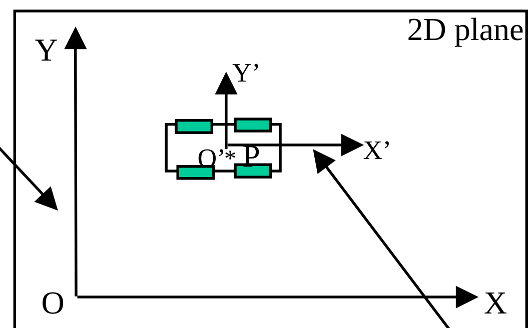
Question 2:

Can we define another coordinate system with its origin being at the point P ?



ANSWER: YES



Definition I: A reference frame is a fixed coordinate system that all objects refer to.

Definition II: An object frame is a coordinate system that is associated to the object itself.

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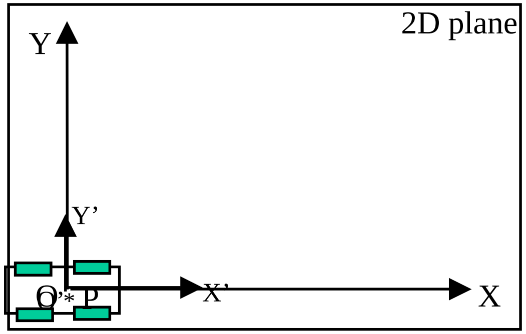
Question 3:

In a 2D space, there is a reference frame for all objects. And, each object has its own object frame. Then, how to describe the relationship between the reference frame and an object frame ?



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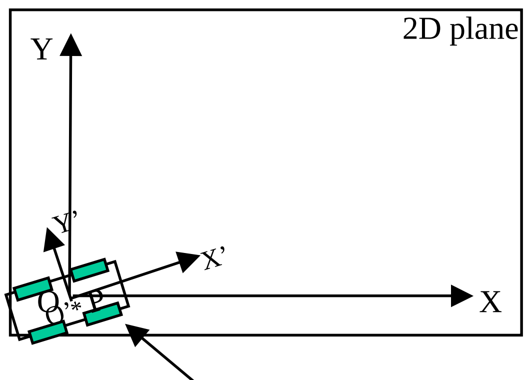
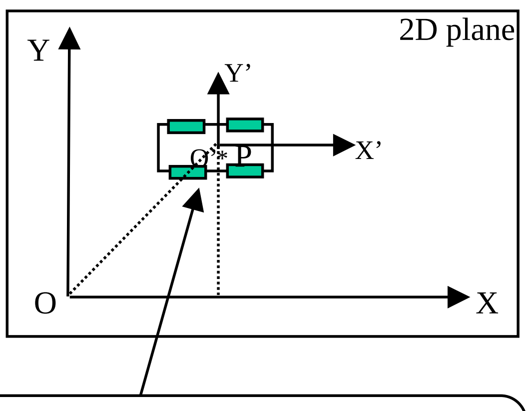
ANSWER:

The relationship is described with two basic transformations:

1. Rotation.
2. Translation.



<u>Rotation:</u> The object frame is rotated around the origin of the reference frame.



<u>Translation:</u> The object frame is displaced from the reference frame's origin to a new position.

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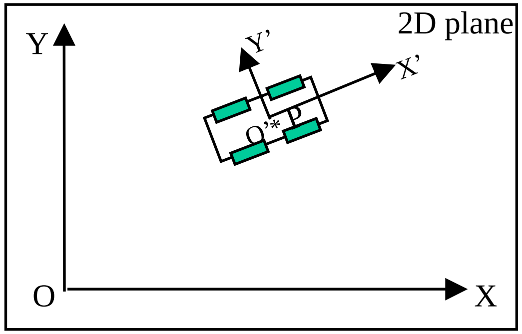
Example:

The general relationship between the reference frame and an object frame can be described by a combination of rotation and translation.



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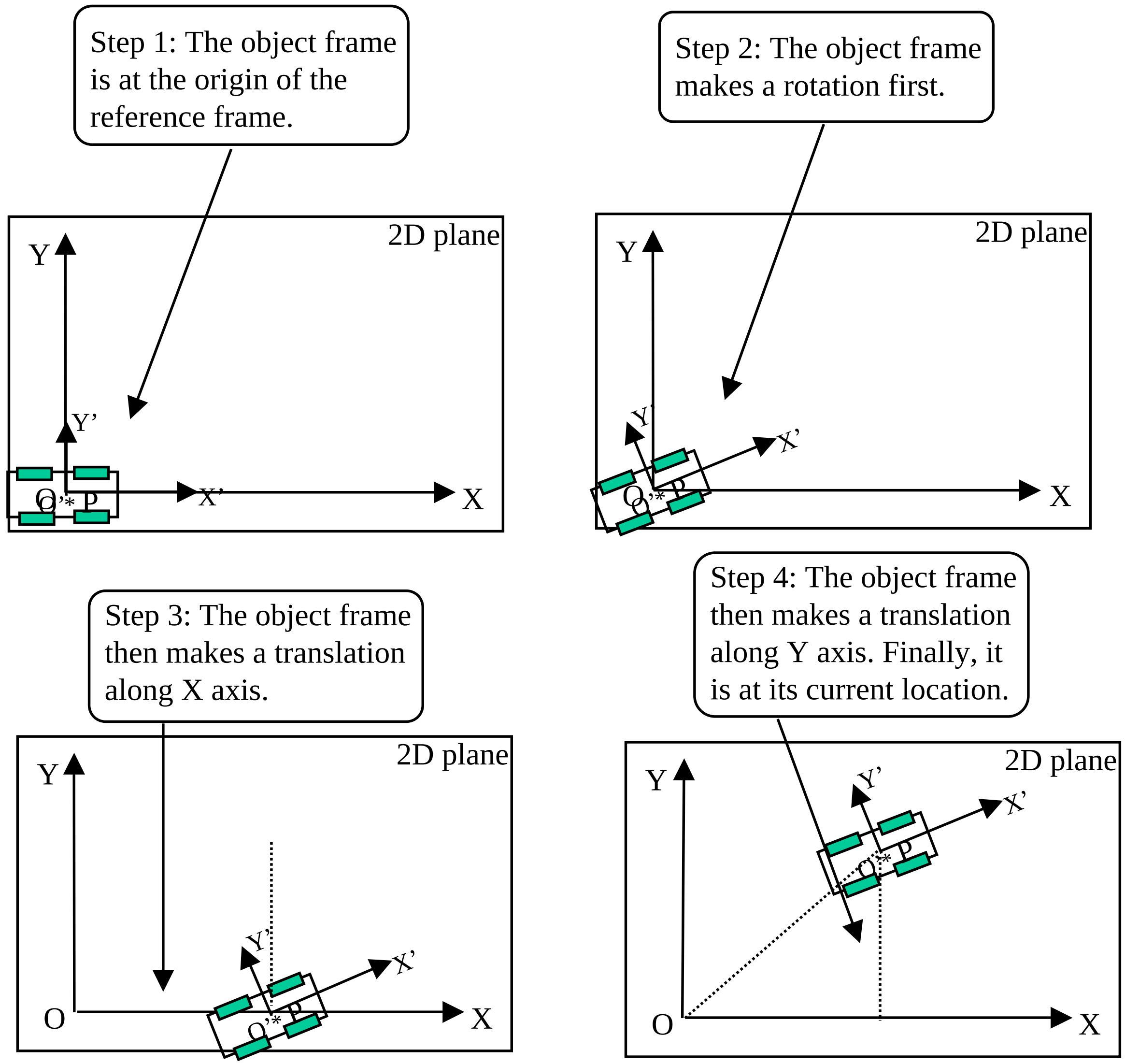

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Question 4:

How to mathematically describe a rotation ?

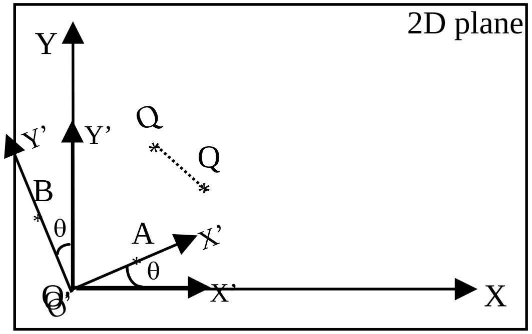
ANSWER:

To use rotation matrix

A rotation matrix is a squared and orthonormal matrix. In a 2D space, a rotation matrix has two column vectors:

(1) The first column vector is the unit vector of the object frame's X` axis in the reference frame (A=(1,0)).

(2) The second column vector is the unit vector of the object frame's Y` axis in the reference frame (B=(0,1)).



$$ {}^{r}R_{o} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} $$

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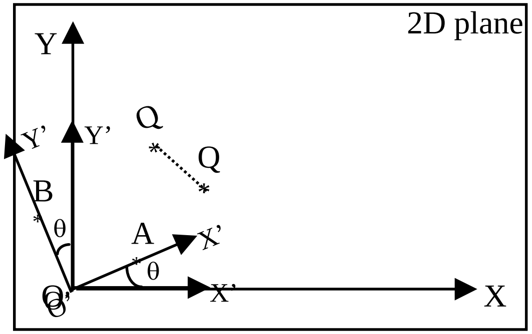
Usage of Rotation Matrix:

In the case of rotation only, the rotation matrix allows to convert the coordinates of a point Q in the object frame into the coordinates of the same point in the reference frame in the following way:

$$ {}^{r}Q = ({}^{r}R_{o}) \bullet ({}^{o}Q) $$

(Coordinates in reference frame = Coordinates in object frame * Rotation)

This operation is called “rotation transformation”.



$$ {}^{r}R_{o} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} $$

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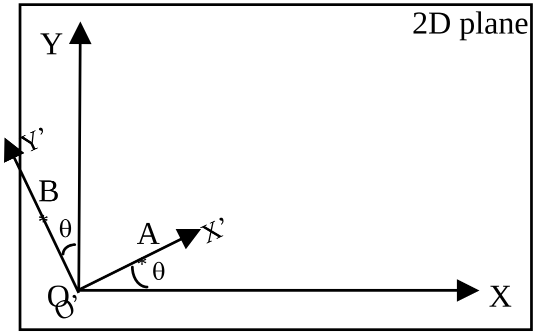
Example:

1. The coordinates of the point A in the object frame are (1,0). What are the coordinates of the point A in the reference frame after the rotation ?

ANSWER:



$$^{o}A = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$^{r}A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

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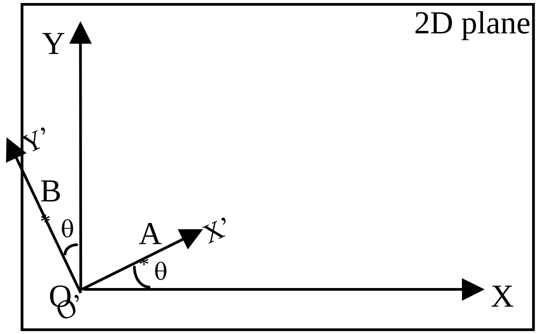
Example:

2. The coordinates of the point B in the object frame are (0,1). What are the coordinates of the point B in the reference frame after the rotation ?

ANSWER:



$$^{o}B = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$^{r}B = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

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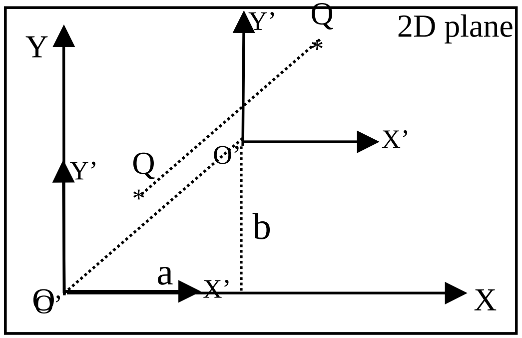
Question 5:

How to mathematically describe a translation ?

ANSWER:

To use translation vector

A translation vector is the position vector of the object frame's origin in the reference frame.



$$^{r}T_{o} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

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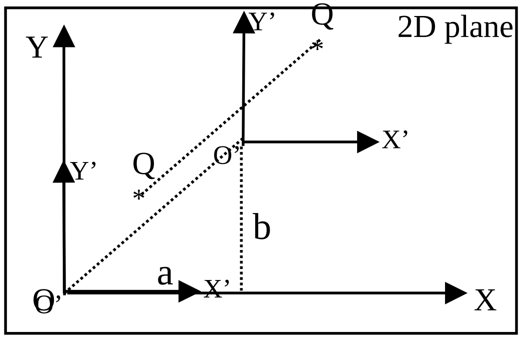
Usage of Translation Vector:

In the case of translation only, the translation vector allows to convert the coordinates of a point Q in the object frame into the coordinates of the same point Q in the reference frame in the following way:

(Coordinates in reference frame = Coordinates in object frame + Translation)

$$^{r}Q = (^{o}Q) + (^{r}T_{o})$$

This operation is called “translation transformation”.



$$^{r}T_{o} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

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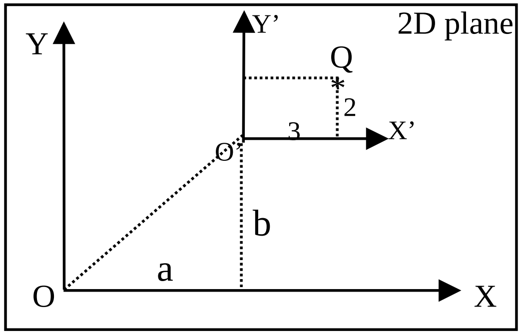
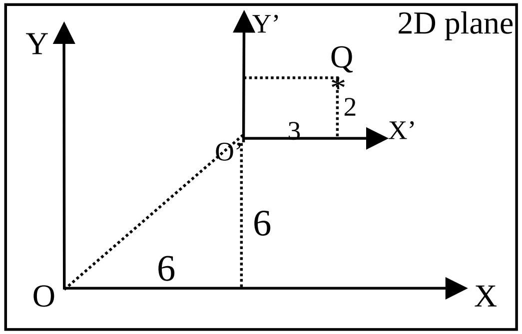
Example:

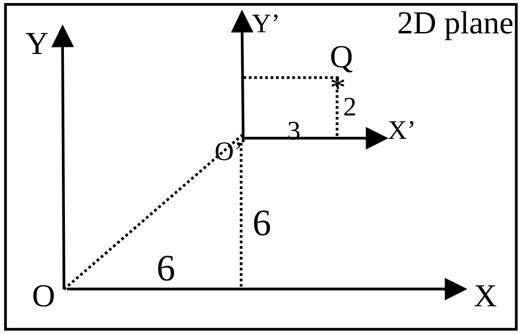
1. The coordinates of a point Q in the object frame are (3,2);



2. The translation vector is (6,6) (a=6 and b=6);



3. What are the coordinates of the same point Q in the reference frame ?



$$^{o}Q = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$^{r}Q = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

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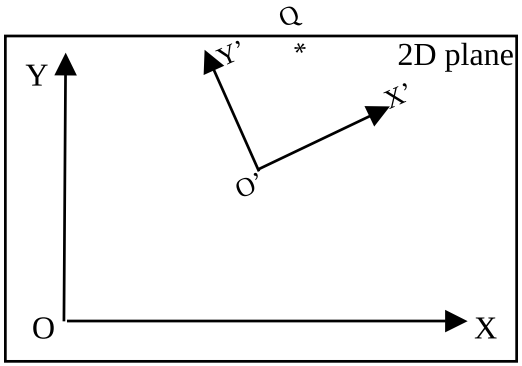
Question 6:

How to mathematically describe the general relationship between an object frame and the reference frame ?



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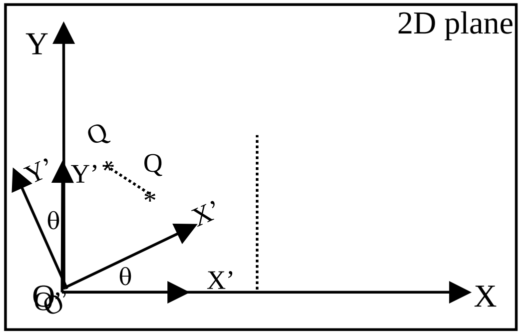
ANSWER:

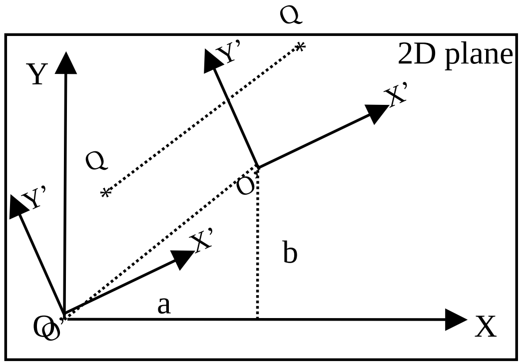
The general relationship between an object frame and the reference frame can be described by a combination of rotation and translation:



(a) Rotation first:

$$^{r}R_{o} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$^{r}Q = (^{r}R_{o}) \bullet (^{o}Q)$$



(b) Translation second:

$$^{r}T_{o} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$^{r}Q = (^{r}R_{o}) \bullet (^{o}Q) + {}^{r}T_{o}$$

(Rotation + Translation) are generally called “motion transformation”

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Question 7:

Can the “motion transformation” be written in a more compact form ? (How to use matrix operation to describe both “rotation” transformation and “translation” transformation ?)

ANSWER: YES

Step 1: Write the “motion transformation” in its original form:

$$\begin{bmatrix} {}^{r}X \\ {}^{r}Y \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \bullet \begin{bmatrix} {}^{o}X \\ {}^{o}Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Coordinates in reference frame

Rotation matrix

Coordinates in object frame

Translation vector

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Step 2: Write compact form by introducing an extra coordinate:

$$\begin{bmatrix} {}^{r}X \\ {}^{r}Y \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} {}^{o}X \\ {}^{o}Y \\ 1 \end{bmatrix}$$

Definition III: The coordinates with an extra element are called “homogenous coordinates”. Two advantages are:
(a) scaling effect in graphics and (b) compact description of motion transformation.

Motion transformation matrix

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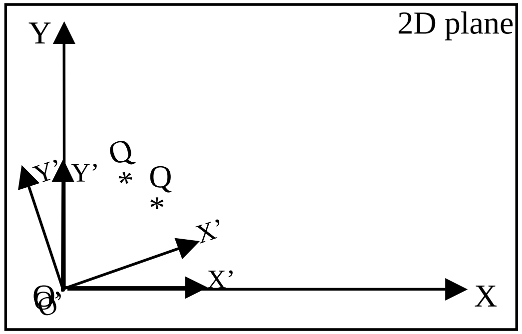
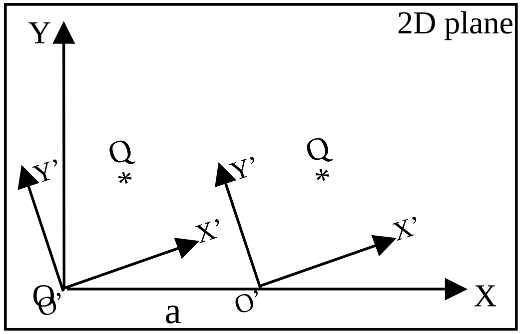
Example:

1. Pure rotation:



$$Rot(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Pure translation along X axis:



$$Trans(x,a) = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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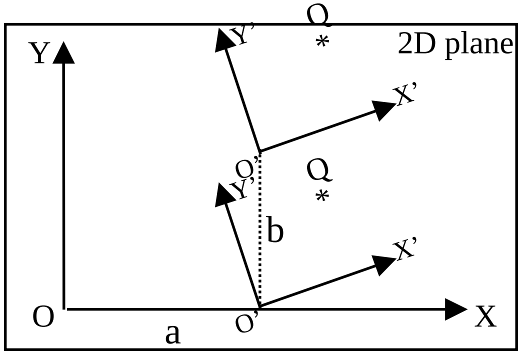
3. Pure translation along Y axis:



$$Trans(y,b) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

4. Combined motion transformation matrix:

$$^{r}M_{o} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & a \\ \sin(\theta) & \cos(\theta) & b \\ 0 & 0 & 1 \end{bmatrix} = Trans(y,b) \bullet Trans(x,a) \bullet Rot(\theta)$$

$$^{r}P = (^{r}M_{o}) \bullet (^{o}P)$$

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SUMMARY

1. In 2D space, a reference frame is needed in order to describe object location.

2. An object can have its own coordinate system that is called “object frame”.

3. The relationship between the reference frame and an object frame is composed of rotation and translation:

$$^{r}M_{o} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & a \\ \sin(\theta) & \cos(\theta) & b \\ 0 & 0 & 1 \end{bmatrix} = Trans(y,b) \bullet Trans(x,a) \bullet Rot(\theta)$$

$$^{r}P = (^{r}M_{o}) \bullet (^{o}P)$$

4. The initial position of an object frame is always assumed to be at the origin of the reference frame.

5. Any point is attached to either the “reference frame” or a “object frame”. Hence, motion of a point is equivalent to the motion of the corresponding frame of the point.

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